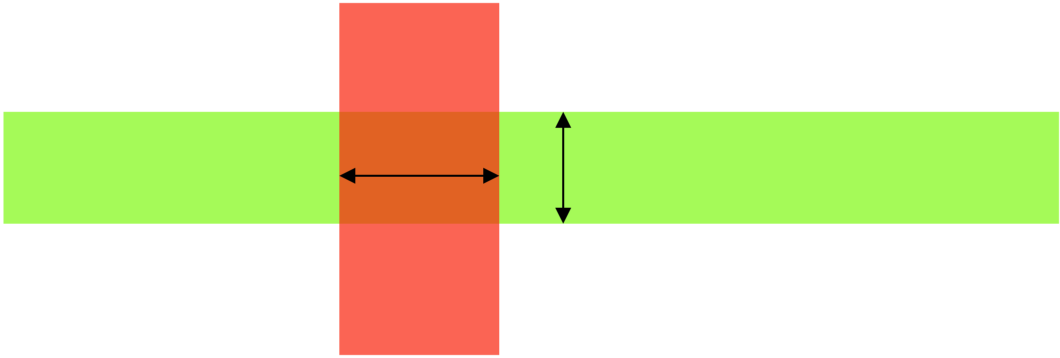
EE 330
Lecture 4

- Yield
- Statistics Review

Review from last lecture:

Feature Size

Feature size is the minimum lateral feature size that can be **reliably** manufactured



Often given as either feature size or pitch

Minimum feature size often identical for different features

Extremely challenging to decrease minimum feature size in a new process

Review from last lecture:

What is meant by "reliably"

Yield is acceptable if circuit performs as designed even when a very large number of these features are made

If P is the probability that a feature is good

n is the number of uncorrelated features on an IC

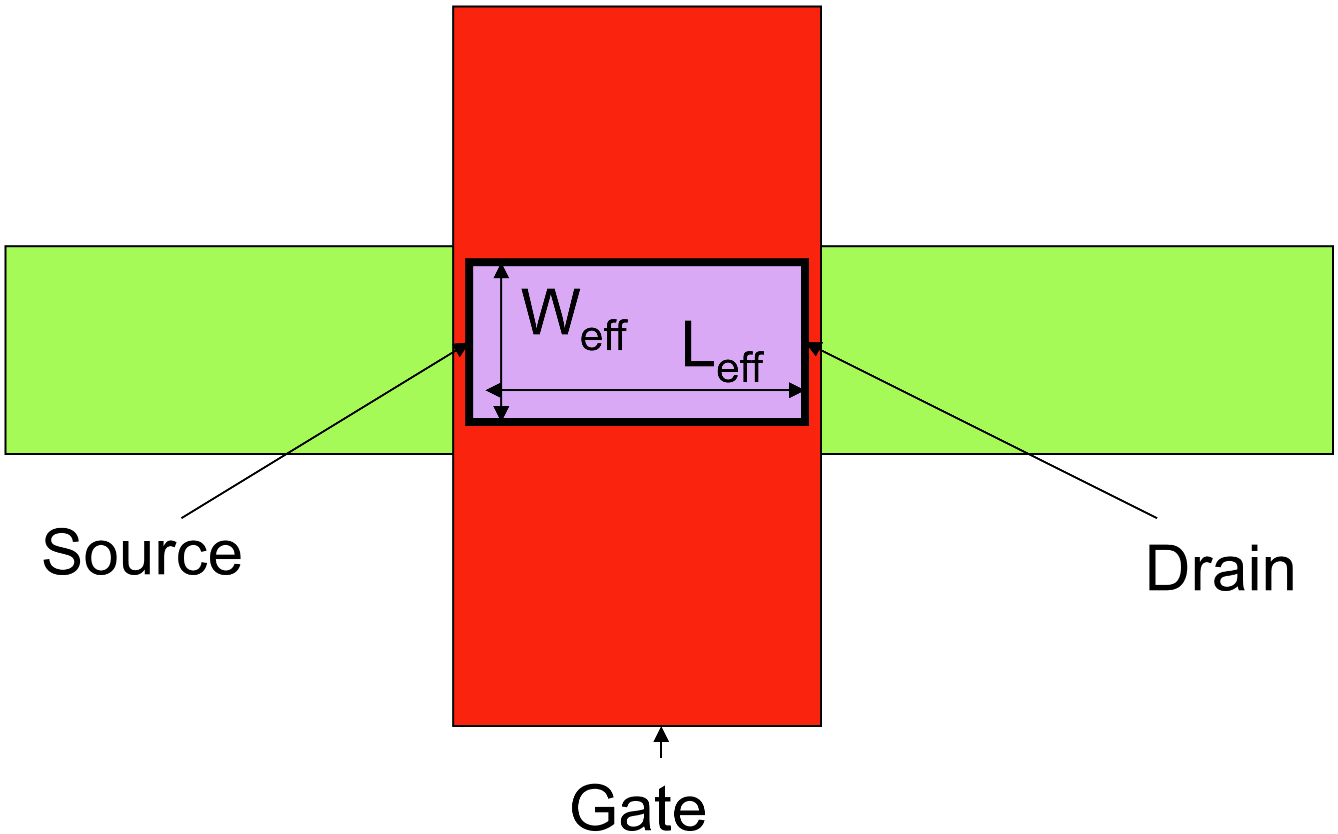
Y is the yield

$$Y = P^n$$

$$P = e^{\frac{\log_e Y}{n}}$$

Review from last lecture:

MOS Transistor



Effective Width and Length Generally Smaller than Drawn Width and Length

Review from last lecture:

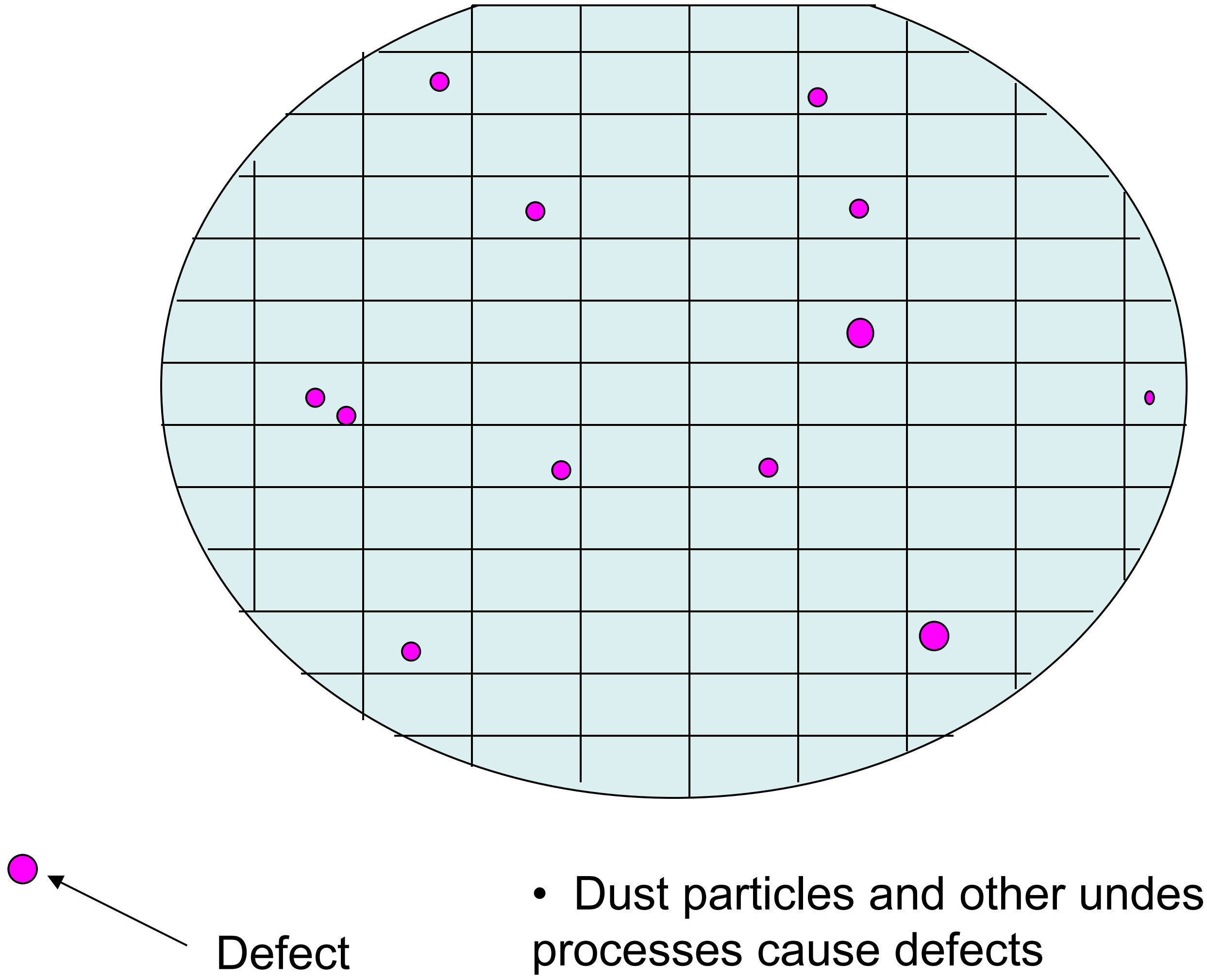
Physical Characteristics of Key Semiconductor Materials

Silicon:	Average Atom Spacing	$2.7\,\overset{o}{A}$
	Lattice Constant	$5.4\,\overset{o}{A}$
S_iO_2	Average Atom Spacing	$3.5\,\overset{o}{A}$
	Breakdown Voltage	$5 \text{ to } 10\text{MV/cm} = 5 \text{ to} 10\text{mV/} \overset{0}{A}$
Air		20KV/cm

Physical size of atoms and molecules place fundamental limit on conventional scaling approaches

Review from last lecture:

Defects in a Wafer



- Dust particles and other undesirable processes cause defects
- Defects in manufacturing cause yield loss

Review from last lecture:

Hard Fault Model

$$Y_H = e^{-Ad}$$

Y_H is the probability that the die does not have a hard fault
A is the die area
d is the defect density
(for some older processes, typically $1\text{cm}^{-2} < d < 2\text{cm}^{-2}$)
for some newer processes, typically $0.1\text{cm}^{-2} < d < 1\text{cm}^{-2}$)

Industry often closely guards the value of d for their process

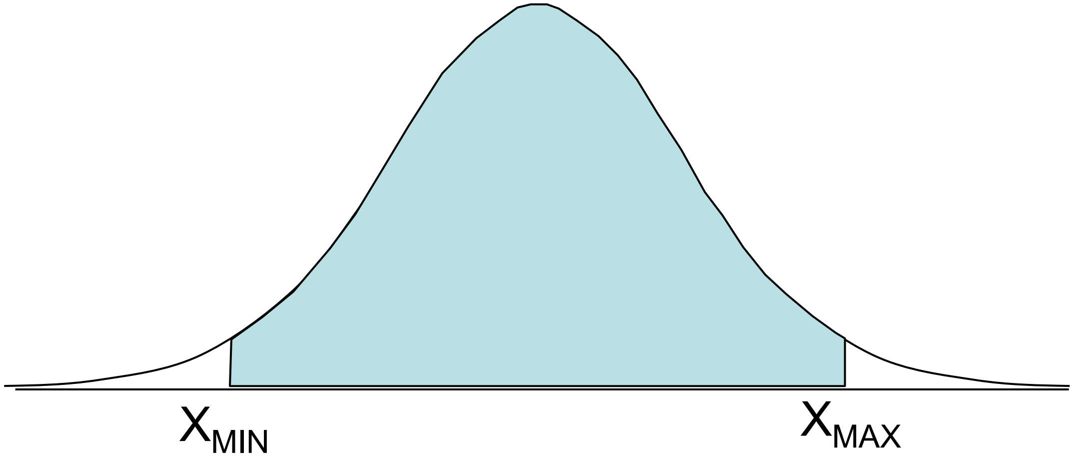
Other models, which may be better, have the same general functional form

Review from last lecture:

Soft Fault Model

$$P_{SOFT} = \int_{X_{MIN}}^{X_{MAX}} f(x)\,dx$$

P_{SOFT} is the soft fault yield
f(x) is the probability density function of the parameter of interest
X_{MIN} and X_{MAX} define the acceptable range of the parameter of interest



Some circuits may have several parameters that must meet performance requirements

Review from last lecture:

Soft Fault Model

If there are k parameters that must meet parametric performance requirements and if the random variables characterizing these parameters are uncorrelated, then the soft yield is given by

$$Y_S = \prod_{j=1}^{k} P_{SOFT_j}$$

Review from last lecture:

Overall Yield

If both hard and soft faults affect the yield of a circuit, the overall yield is given by the expression

$$Y = Y_H Y_S$$

Review from last lecture:

Cost Per Good Die

The manufacturing costs per good die is given by

$$C_{Good} = \frac{C_{FabDie}}{Y}$$

where C_{FabDie} is the manufacturing costs of a fab die and Y is the yield

There are other costs that must ultimately be included such as testing costs, engineering costs, packaging costs, etc.

Do you like statistics ?

Statistics are Real!

Statistics govern what really happens throughout much of the engineering field!

Statistics are your Friend !!!!

- You might as well know what will happen since statistics characterize what WILL happen in the presence of variability in many processes !

Statistics Review

Assume x is a random variable of interest

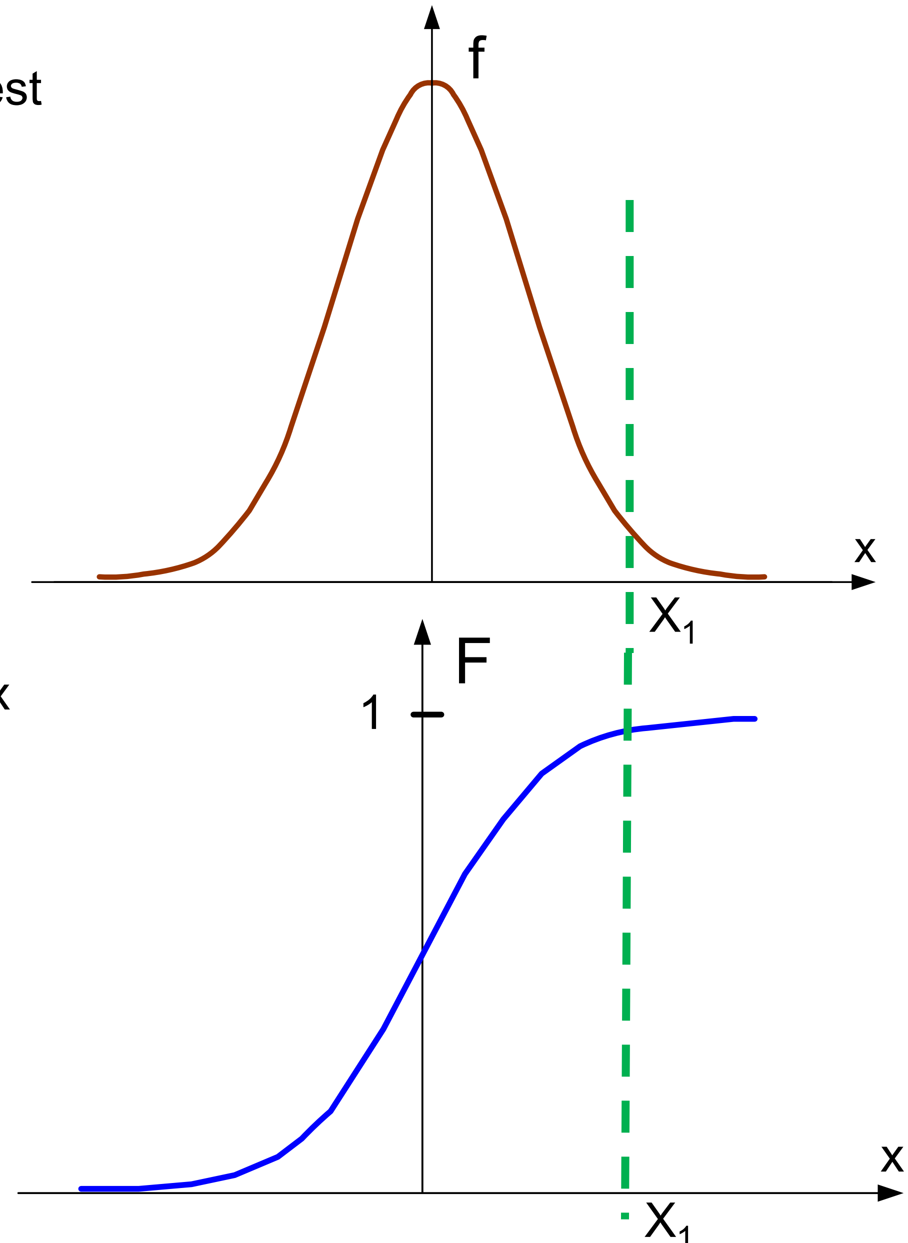
f(x) = Probability Density Function for x

$$\int_{x=-\infty}^{\infty} f(x)\,dx = 1$$

F(x) = Cumulative Density Function for x

$$F(X_1) = \int_{x=-\infty}^{X_1} f(x)\,dx$$

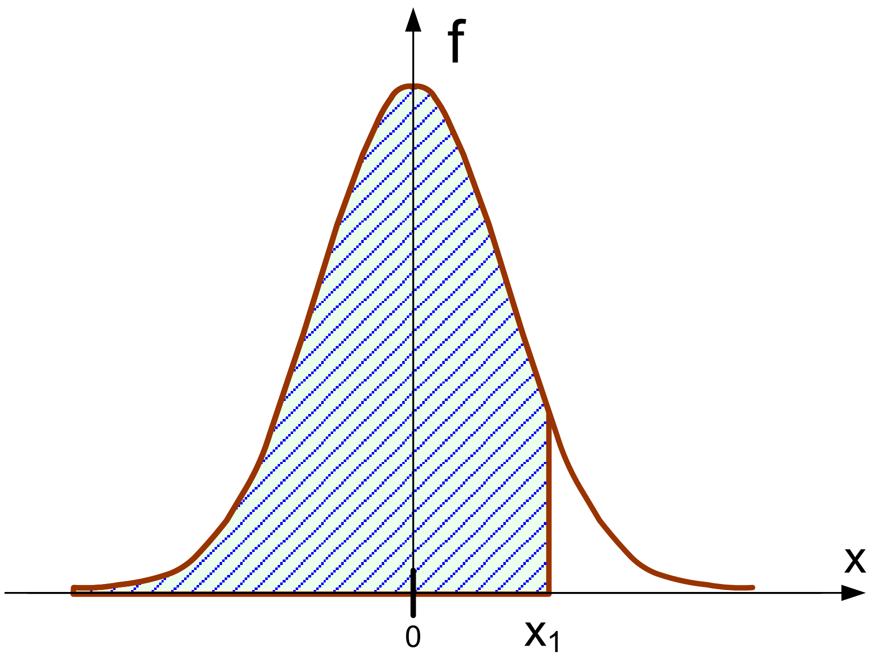
$$0 \le F(x) \le 1 \qquad \frac{\partial F(x)}{\partial x} \ge 0$$

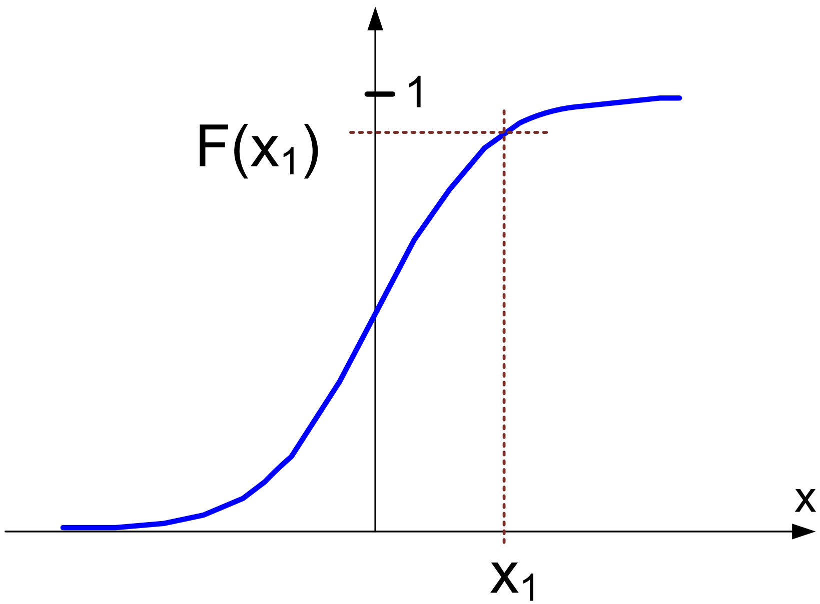


Statistics Review

f(x) = Probability Density Function for x

F(x) = Cumulative Density Function for x





$$P\{x \le x_1\} = \int_{x=-\infty}^{x_1} f(x)\,dx$$

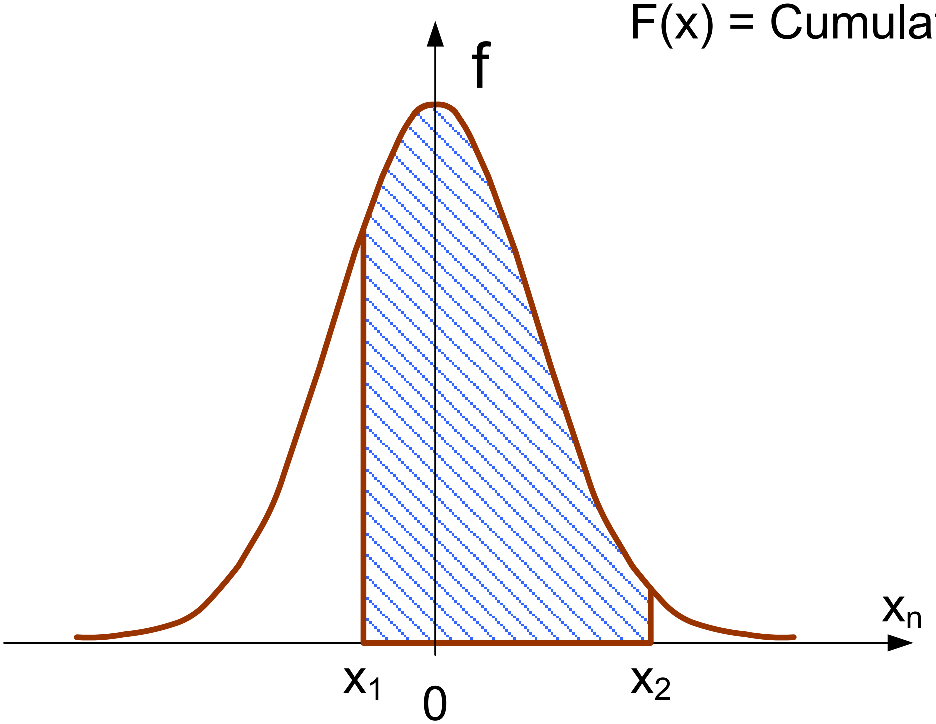
$$P\{x \le X_1\} = F(X_1)$$

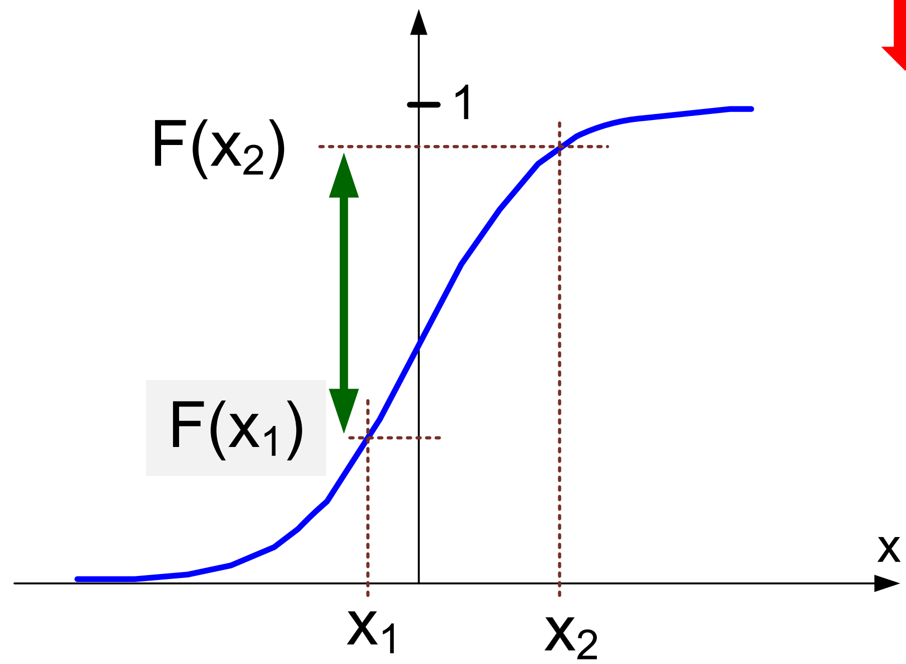
$$F(x_1) = \int_{x=-\infty}^{x_1} f(x)\,dx$$

Statistics Review

f(x) = Probability Density Function for x

F(x) = Cumulative Density Function for x

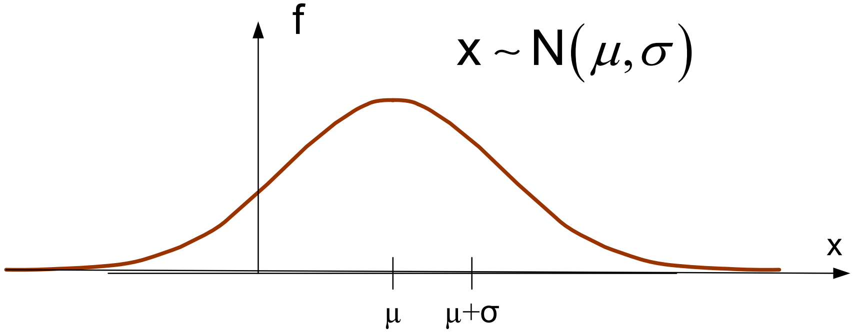


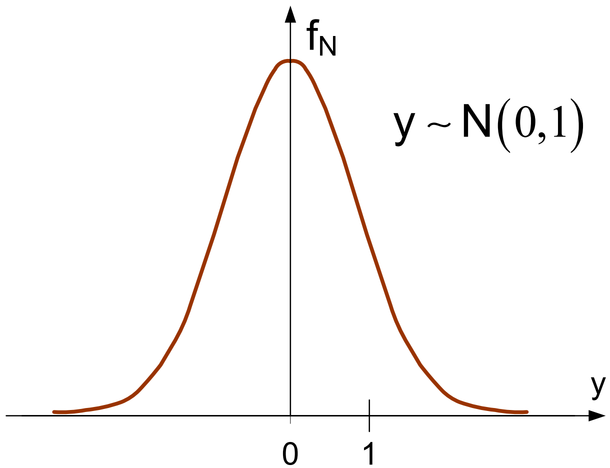


$$P\{X_1 \le x \le X_2\} = \int_{X_1}^{X_2} f(x)\,dx$$

$$P\{X_1 \le x \le X_2\} = F(X_2) - F(X_1)$$

Statistics Review



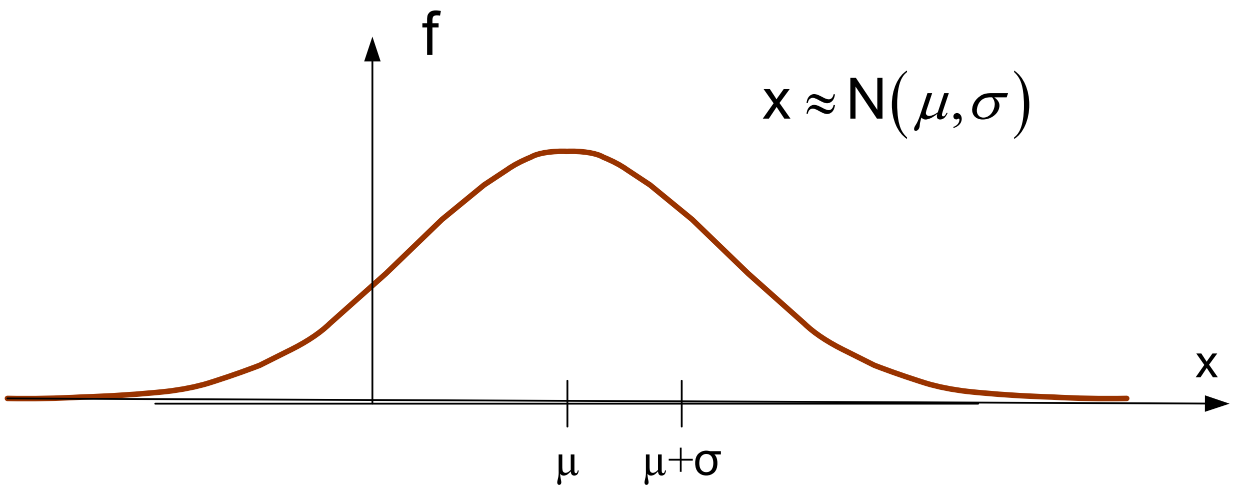


$$\int_{x=-\infty}^{\infty} f(x)\,dx = 1 \qquad y = \frac{x-\mu}{\sigma} \qquad \int_{y=-\infty}^{\infty} f_N(y)\,dy = 1$$

Theorem 1: If the random variable x is normally distributed with mean μ and standard deviation σ, then $y = \frac{x-\mu}{\sigma}$ is also a random variable that is normally distributed with mean 0 and standard deviation of 1.

(Normal Distribution and Gaussian Distribution are the same)

Statistics Review

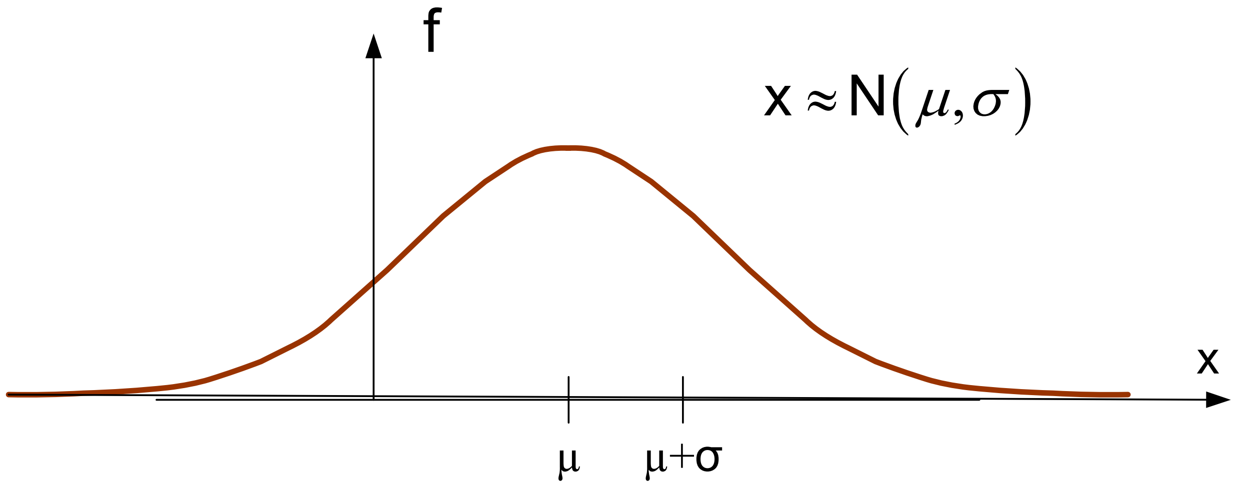


The random part of many parameters of microelectronic circuits is often assumed to be Normally distributed and experimental observations confirm that this assumption provides close agreement between theoretical and experimental results

The mapping $y = \dfrac{x-\mu}{\sigma}$ is often used to simplify the statistical characterization of the random parameters in microelectronic circuits

x generally is dimensioned, y is dimensionless

Statistics Review



$x \approx N(\mu, \sigma)$

Example:

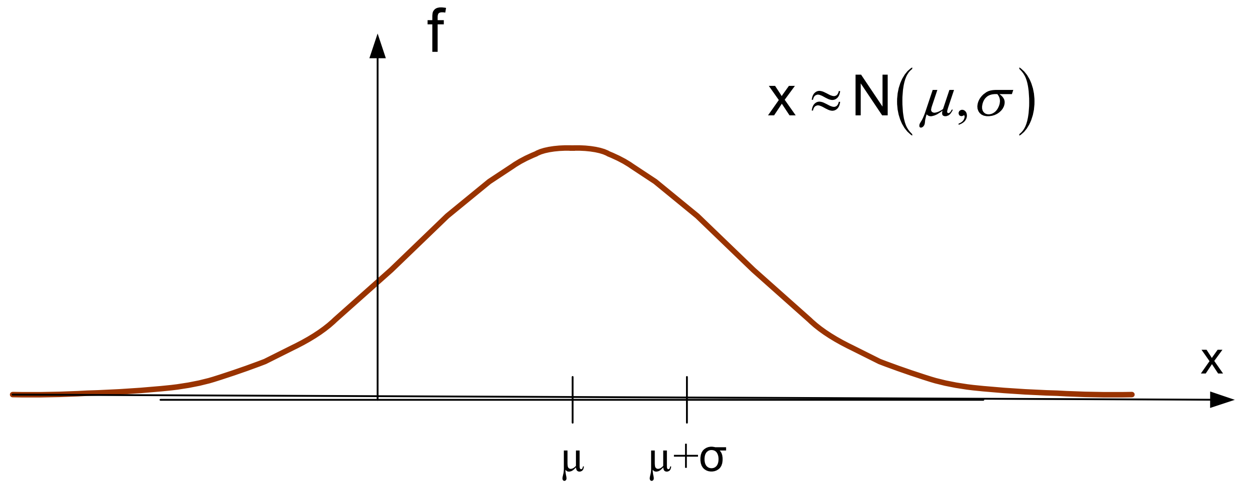
x might be the frequency of oscillation of a ring oscillator used for a clock in a crystal-less digital circuit, x Gaussian (Normal)

Dimensions of x : Hz
Maybe μ=550 MHz σ=50 MHz

$$y = \frac{x - \mu}{\sigma}$$ is dimensionless with $\mu_y = 0$ $\sigma_y = 1$

y: N(0,1)

Statistics Review



Example:

x might be the offset voltage of an op amp, x Gaussian (Normal)

Dimensions of x : Volts
Typically μ=0V σ=10 mV

$$y = \frac{x-\mu}{\sigma}$$ is dimensionless with $\mu_y=0$ $\sigma_y=1$

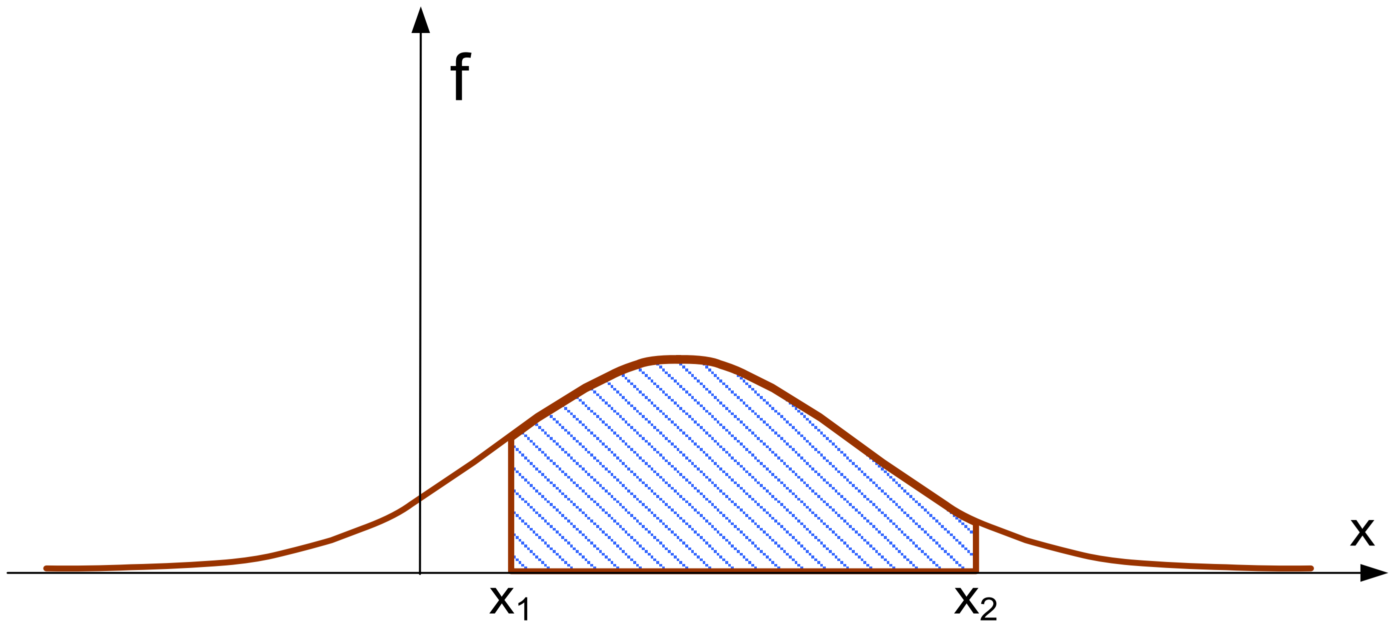
y: N(0,1)

Background Information

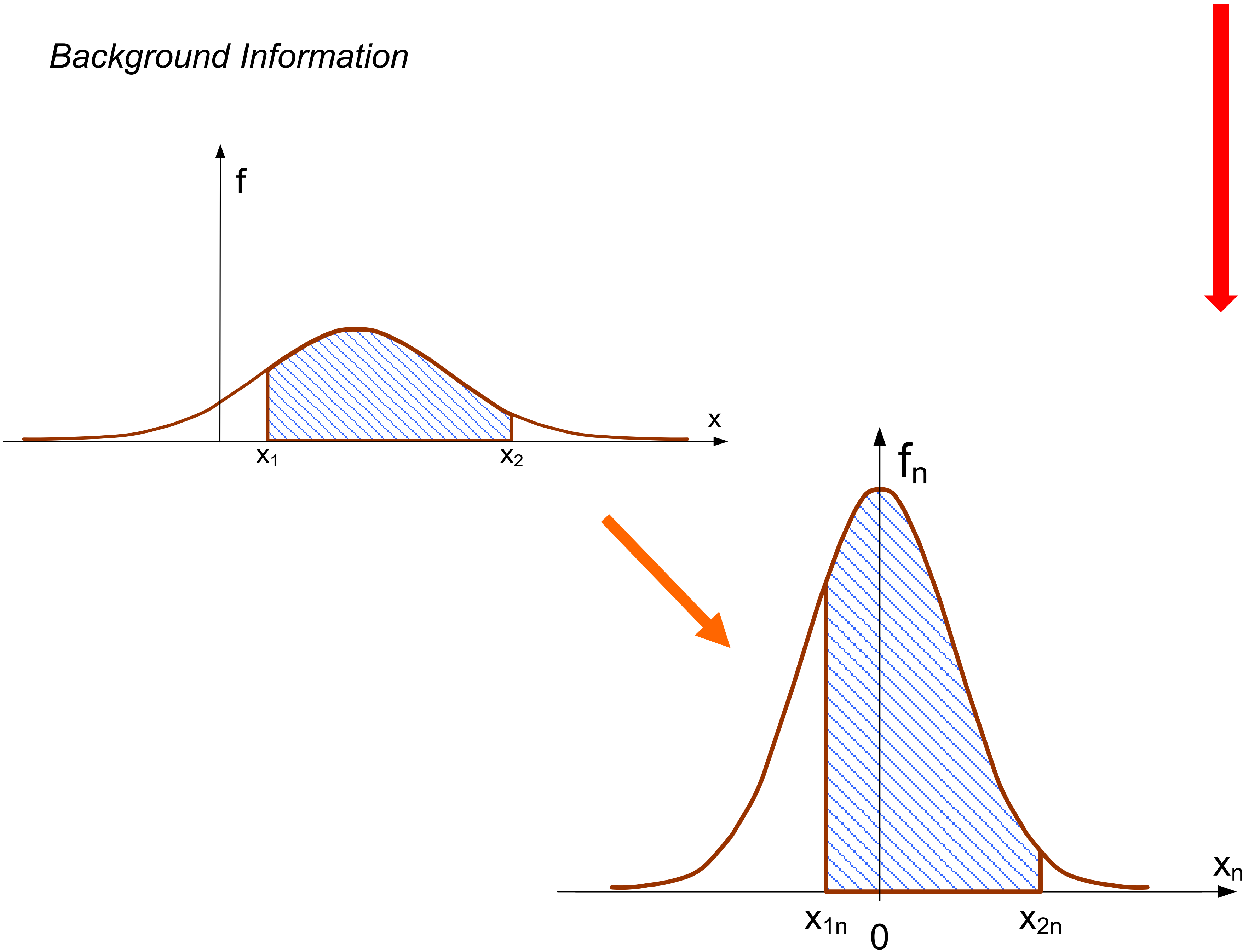
Theorem 2: If x is a Normal (Gaussian) random variable with mean μ and standard deviation σ, then the probability that x is between x_1 and x_2 is given by

$$p = \int_{x_1}^{x_2} f(x)\,dx = \int_{x_{1n}}^{x_{2n}} f_n(x)\,dx \quad \text{where} \quad x_{1n} = \frac{x_1 - \mu}{\sigma} \quad \text{and} \quad x_{2n} = \frac{x_2 - \mu}{\sigma}$$

and where $f_n(x)$ is N(0,1)



Background Information

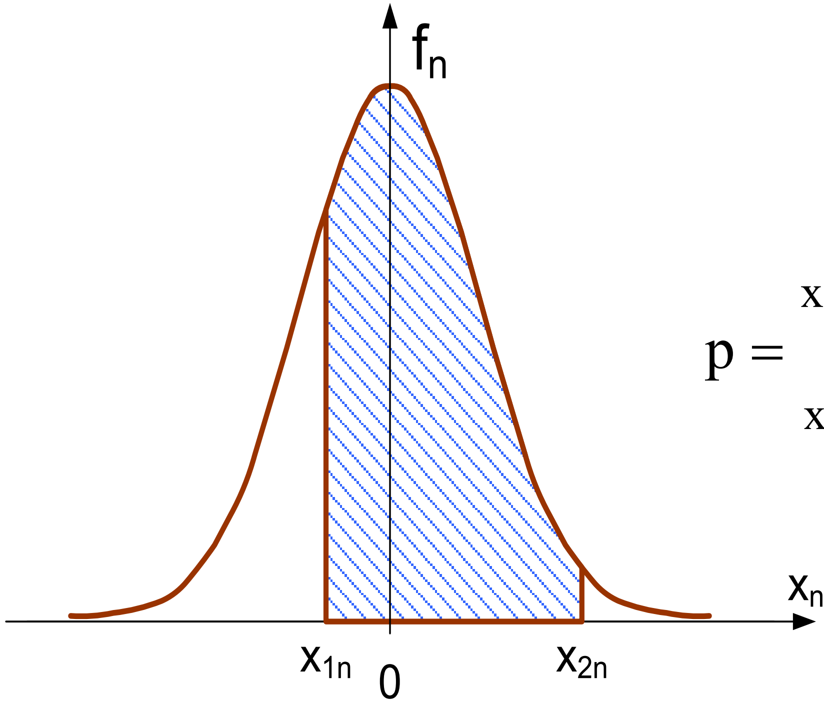


Background Information

Observation: The probability that the N(0,1) random variable x_n satisfies the relationship $x_{1n}<x_n<x_{2n}$ is also given by

$$p = F_n\left(x_{2n}\right) - F_n\left(x_{1n}\right)$$

where $F_n(x)$ is the CDF of x_n.



$$p = \int_{x_{1n}}^{x_{2n}} f_n\left(x\right)dx$$

Since the N(0,1) distribution is symmetric around 0, p can also be expressed as

$$p = F_n\left(x_{2n}\right) - \left(1 - F_n\left(-x_{1n}\right)\right)$$

Background Information

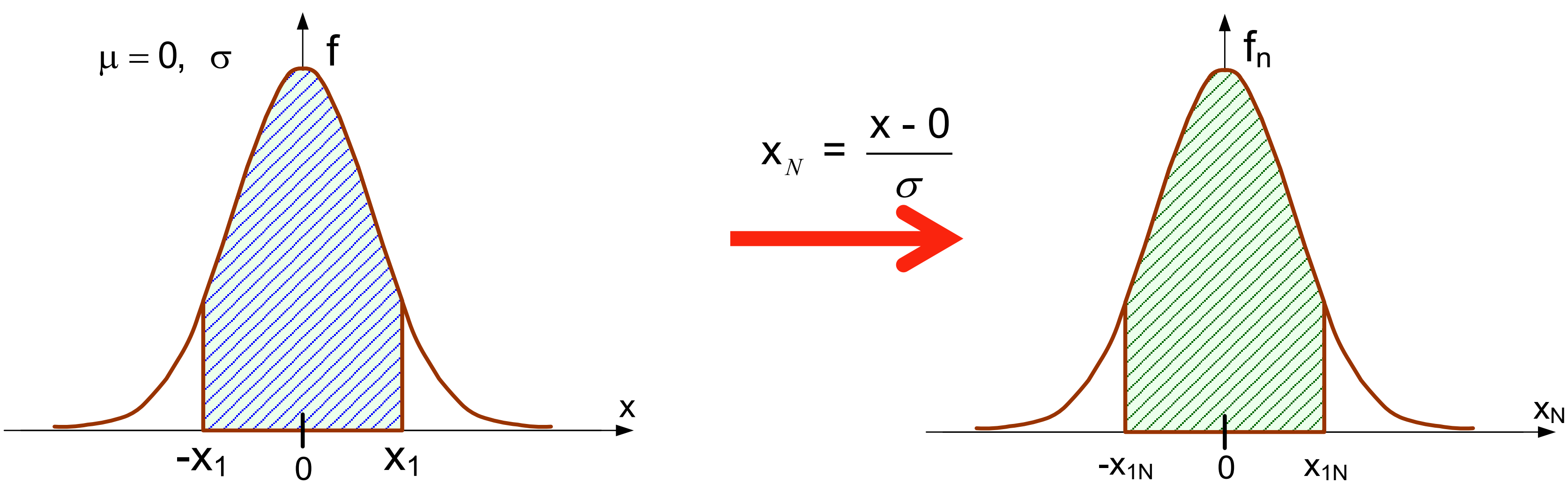
Observation: In many electronic circuits, a random variable of interest, x, is 0 mean Gaussian and the probabilities of interest are characterized by a region defined by the magnitude of the random variable (i.e. $-x_1 < x < x_1$).

In these cases, if we define $x_N = \frac{x - 0}{\sigma}$ then x_N is N(0,1) and

$$p(-x_1 < x < x_1) = \int_{-x_1}^{x_1} f(x)dx = \int_{-x_{1n}}^{x_{1n}} f_n(x)dx = F_n(x_{1n}) - F_n(-x_{1n})$$

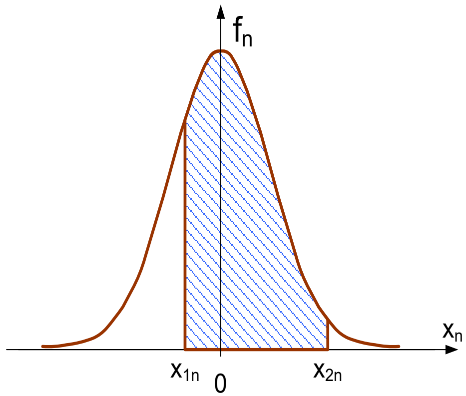
But for the N(0,1) distribution $F_n(-x_{1n}) = 1 - F_n(x_{1n})$

therefore: $p = 2F_n(x_{1n}) - 1$

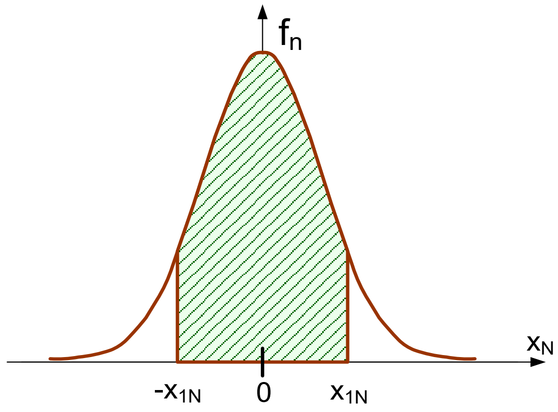


Background Information

$$p = F_n(x_{2n}) - F_n(x_{1n})$$



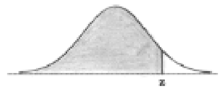
$$p = 2F_n(x_{1n}) - 1$$



Regardless of whether Gaussian performance requirements are asymmetric or symmetric, the CDF of the N(0,1) distribution (i.e. $F_n(x_n)$) can be used to characterize yield

Background Information

Tables of the CDF of the N(0,1) random variable are readily available. It is also available in Matlab, Excel, and a host of other sources.



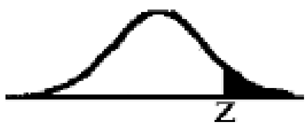
Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

http://www.math.unb.ca/~knight/utility/NormTble.htm

Background Information

Tables of the CDF of the N(0,1) random variable are readily available. It is also available in Matlab, Excel, and a host of other sources.



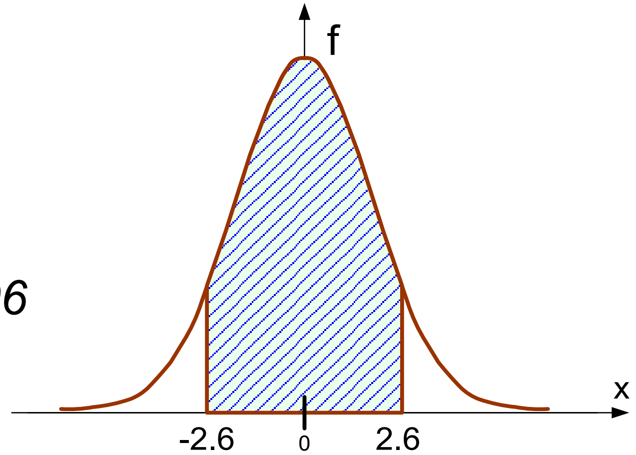
Far Right Tail Probabilities

Z	P{Z to oo}	Z	P{Z to oo}	Z	P{Z to oo}	Z	P{Z to oo}
2.0	0.02275	3.0	0.001350	4.0	0.00003167	5.0	2.867 E-7
2.1	0.01786	3.1	0.0009676	4.1	0.00002066	5.5	1.899 E-8
2.2	0.01390	3.2	0.0006871	4.2	0.00001335	6.0	9.866 E-10
2.3	0.01072	3.3	0.0004834	4.3	0.00000854	6.5	4.016 E-11
2.4	0.00820	3.4	0.0003369	4.4	0.000005413	7.0	1.280 E-12
2.5	0.00621	3.5	0.0002326	4.5	0.000003398	7.5	3.191 E-14
2.6	0.004661	3.6	0.0001591	4.6	0.000002112	8.0	6.221 E-16
2.7	0.003467	3.7	0.0001078	4.7	0.000001300	8.5	9.480 E-18
2.8	0.002555	3.8	0.00007235	4.8	7.933 E-7	9.0	1.129 E-19
2.9	0.001866	3.9	0.00004810	4.9	4.792 E-7	9.5	1.049 E-21

Background Information

Example: Determine the probability that the N(0,1) random variable has magnitude less than 2.6

$$p = 2F_n(2.6) - 1$$



From the table of the CDF, $F_n(2.6) = 0.9953$ *so* $p = .9906$

Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
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1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
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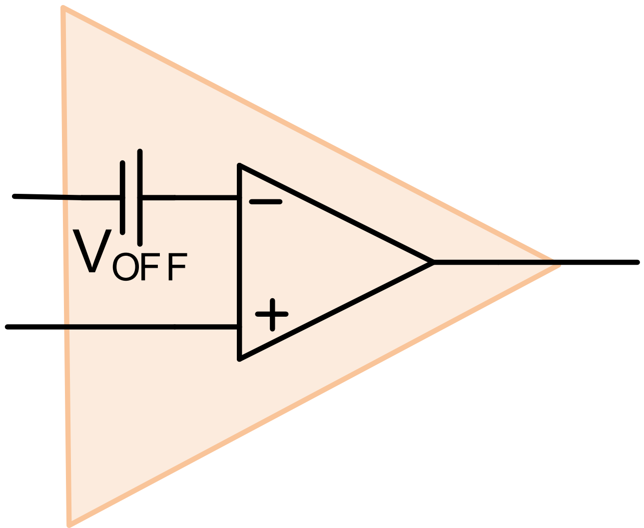
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	[illegible]	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	[illegible]	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

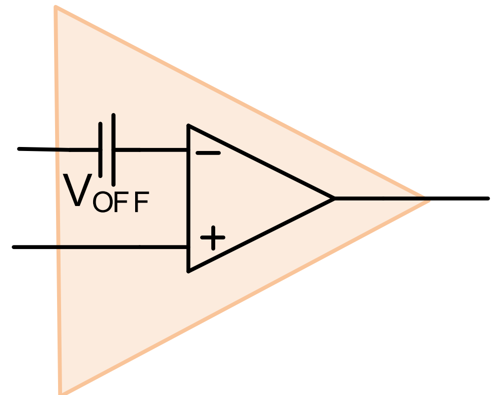
Background Information



- Offset voltage of op amp can be modeled as a dc voltage source in series with input
- Offset voltage is a random variable – usually zero mean and Gaussian
- Often characterized by its standard deviation
- Designer has control of offset through architecture and device sizing
- Invariably low offset voltages require large area

Background Information

Offset Voltage:



TL081, TL081A, TL081B, TL081H
TL082, TL082A, TL082B, TL082H
TL084, TL084A, TL084B, TL084H

TEXAS INSTRUMENTS

SLOS081N – FEBRUARY 1977 – REVISED JUNE 2024

TL08xx FET-Input Operational Amplifiers

1 Features

- High slew rate: 20V/µs (TL08xH, typ)
- Low offset voltage: 1mV (TL08xH, typ)
- Low offset voltage drift: 2 µV/°C
- Low power consumption: 940µA/ch (TL08xH, typ)
- Wide common-mode and differential voltage ranges
  - Common-mode input voltage range includes V_{CC+}
- Low input bias and offset currents
- Low noise: $V_n = 18nV/\sqrt{Hz}$ (typ) at f = 1kHz
- Output short-circuit protection
- Low total harmonic distortion: 0.003% (typ)
- Wide supply voltage: ±2.25V to ±20V, 4.5V to 40V

3 Description

The TL08xH (TL081H, TL082H, and TL084H) family of devices are the next-generation versions of the industry-standard TL08x (TL081, TL082, and TL084) devices. These devices provide outstanding value for cost-sensitive applications, with features including low offset (1mV, typical), high slew rate (20V/µs), and common-mode input to the positive supply. High ESD (1.5kV, HBM), integrated EMI and RF filters, and operation across the full –40°C to 125°C enable the TL08xH devices to be used in the most rugged and demanding applications.

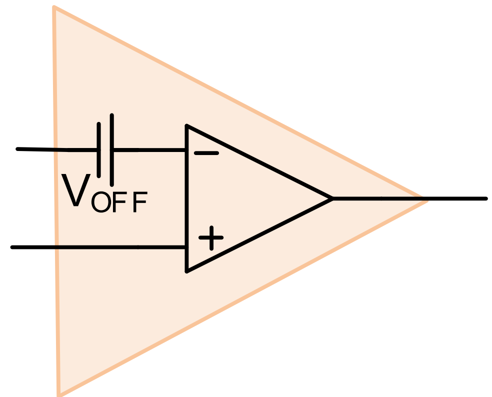
Device Information

PART NUMBER	PACKAGE(1)	BODY SIZE (NOM)(2)
	P (PDIP, 8)	9.59mm × 6.35mm

But read the fine print !

Background Information

Offset Voltage:



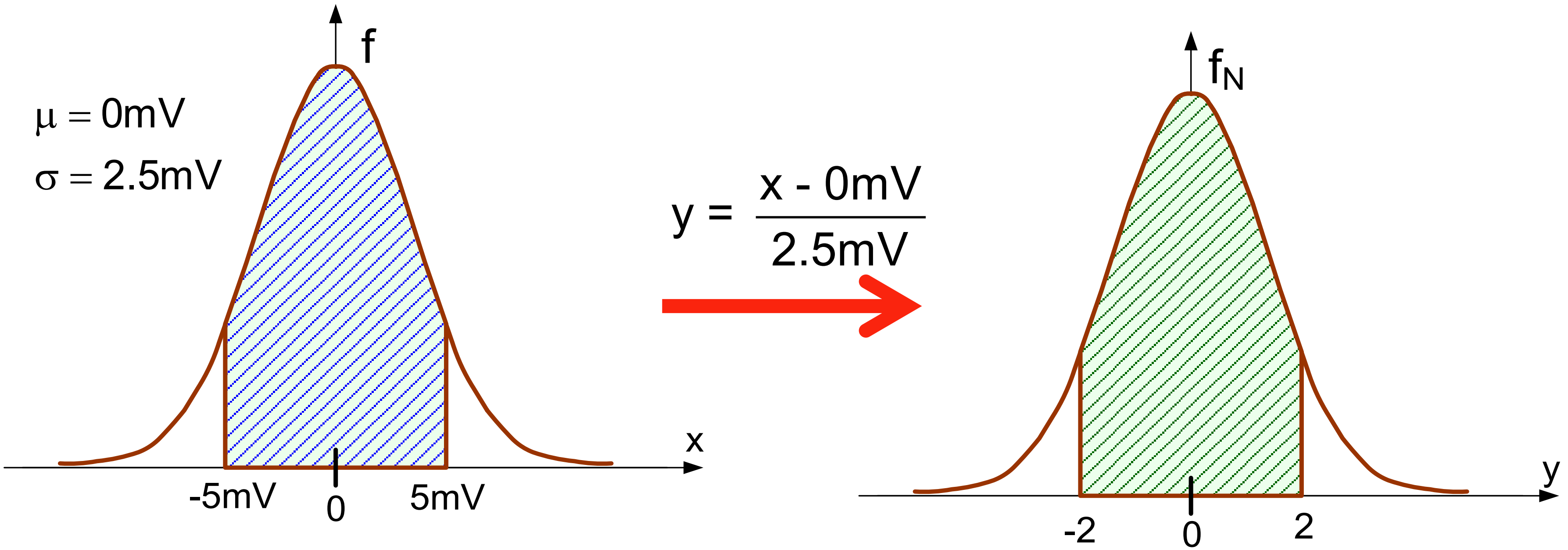
5.8 Electrical Characteristics (DC): TL08xC, TL08xAC, TL08xBC, TL08xI, TL08xM

For $V_S = (V_{CC+}) - (V_{CC-}) = \pm15V$ at $T_A = 25°C$, unless otherwise noted

PARAMETER		TEST CONDITIONS[(1) (2)]			MIN	TYP	MAX	UNIT
V_{OS}	Input offset voltage	$V_O = 0V$ $R_S = 50\ \Omega$	TL08xC			3	10	mV
				T_A = Full range			13	
			TL08xAC			3	6	
				T_A = Full range			7.5	
			TL08xBC			2	3	
				T_A = Full range			5	
			TL08xI			3	6	
				T_A = Full range			8	
			TL081M, TL082M			3	6	
				T_A = Full range			9	
			TL084M			3	9	
				T_A = Full range			15	

At manufacture, V_{OFF} is a random variable and the TL081M has been sorted at test to cut off tails beyond $\pm$ 9mV

Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 2.5mV and a mean of 0V.

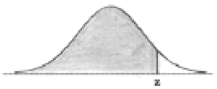


$$p = \int_{-2}^{2} f_N(x)\,dx = F_N(2) - F_N(-2) = 2 * F_N(2) - 1$$

$$p = 2 * F_N(2) - 1$$

Background Information

Example (continued)



Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
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2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
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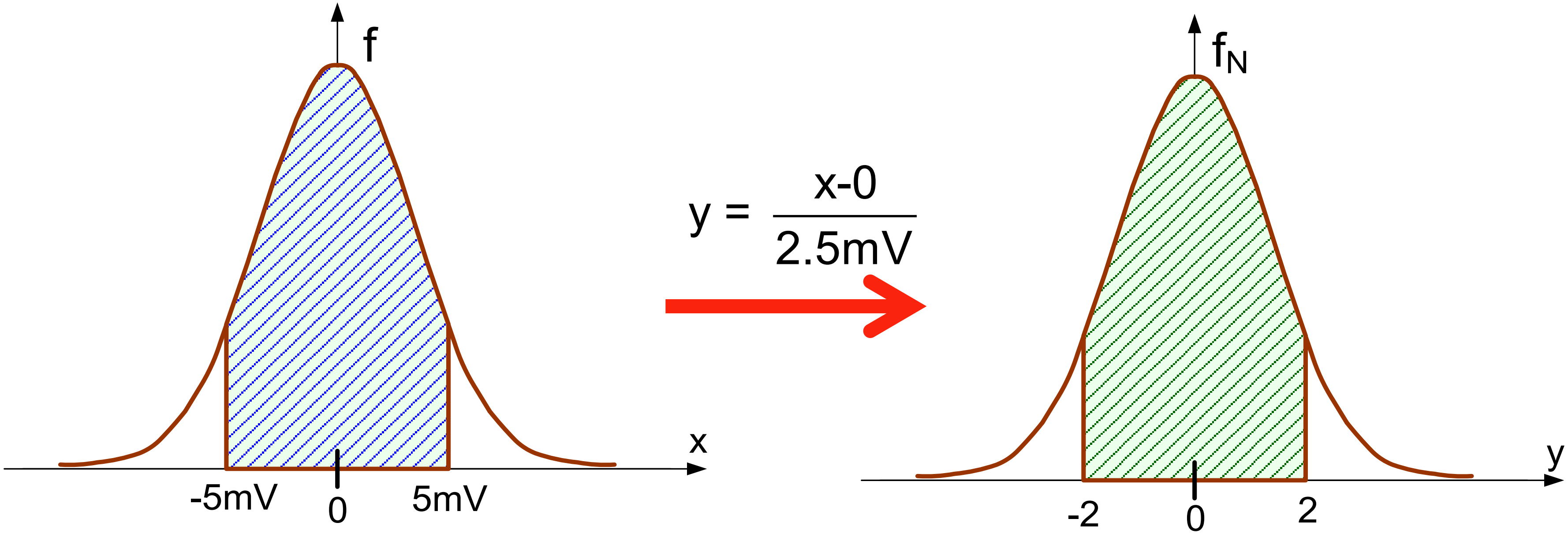
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

http://www.math.unb.ca/~knight/utility/NormTble.htm

Example (continued)

Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 2.5mV and a mean of 0V.



$$p = 2 * F_N(2) - 1$$

$$F_N(2) = 0.9772$$

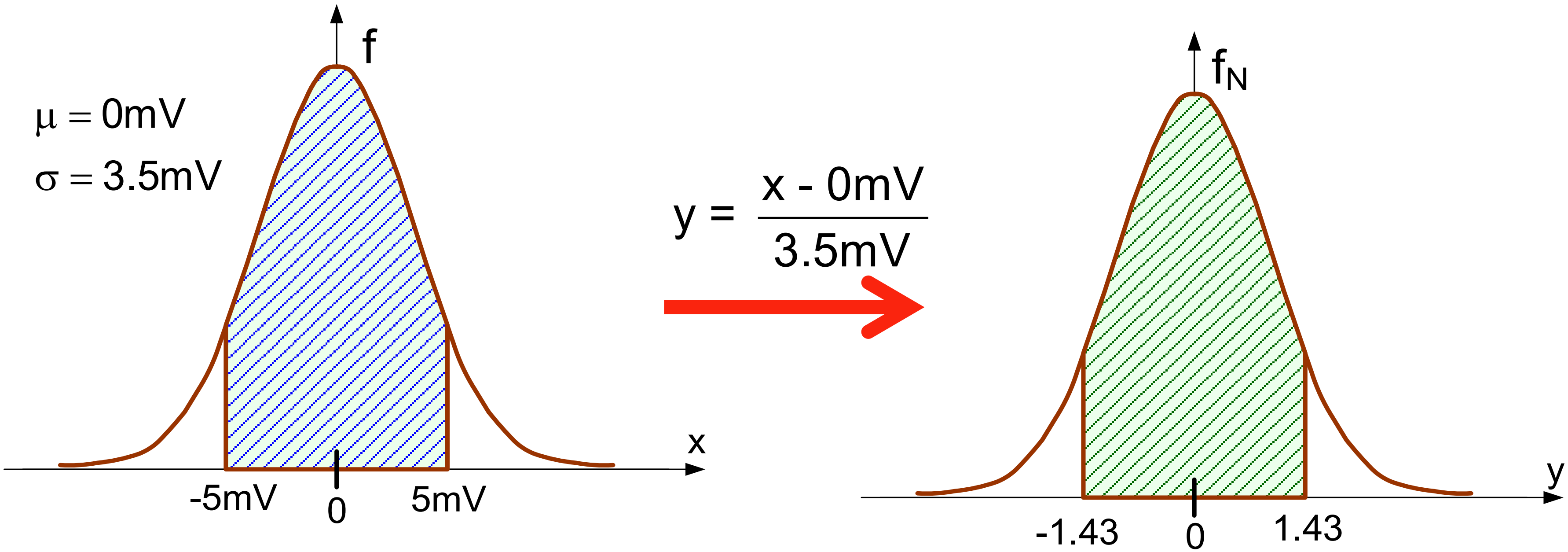
$$p = 2 * .9772 - 1 = .9544$$

$$Y_{SOFT} = 95.4\%$$

Background Information

Repeat the previous example if the designer decided to reduce the area so that the standard deviation increased to 3.5 mV

Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 3.5mV and a mean of 0V.

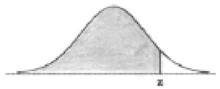


$$p = \int_{-1.43}^{1.43} f_N(x)\,dx = F_N(1.43) - F_N(-1.43) = 2 * F_N(1.43) - 1$$

$$p = 2 * F_N(1.43) - 1$$

Background Information

Example (continued)



Probability Content
from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
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0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
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1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
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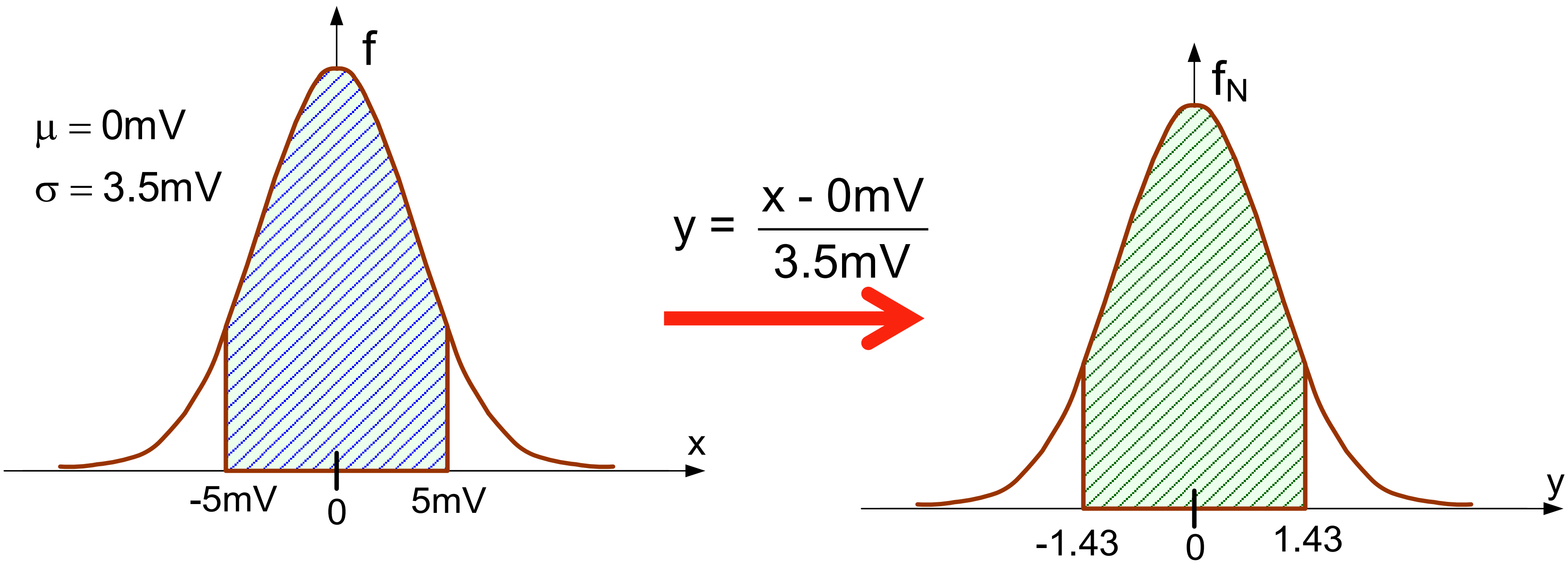
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

http://www.math.unb.ca/~knight/utility/NormTble.htm

Background Information

Repeat the previous example if the designer decided to reduce the area so that the standard deviation increased to 3.5 mV

Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 3.5mV and a mean of 0V.

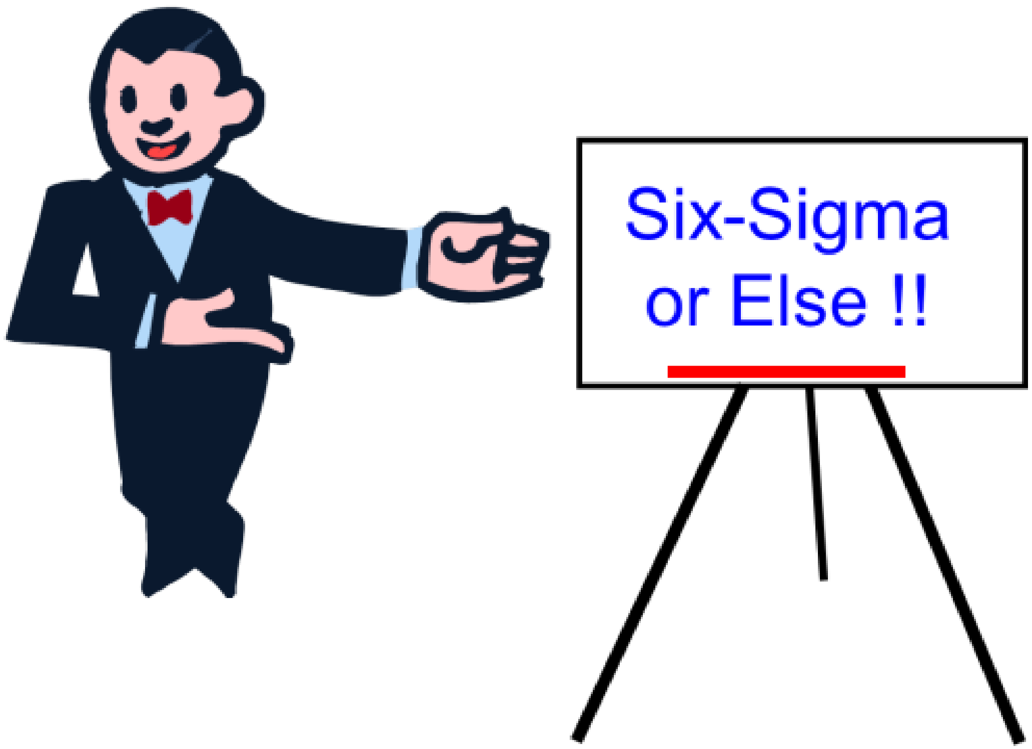


$$p = 2 * F_N(1.43) - 1 = 2 * 0.9236 - 1 = 0.847$$

This small change in the design dropped the yield from just over 95% to just under 85%

Statistical analysis is critical for predicting performance capabilities of many ICs !

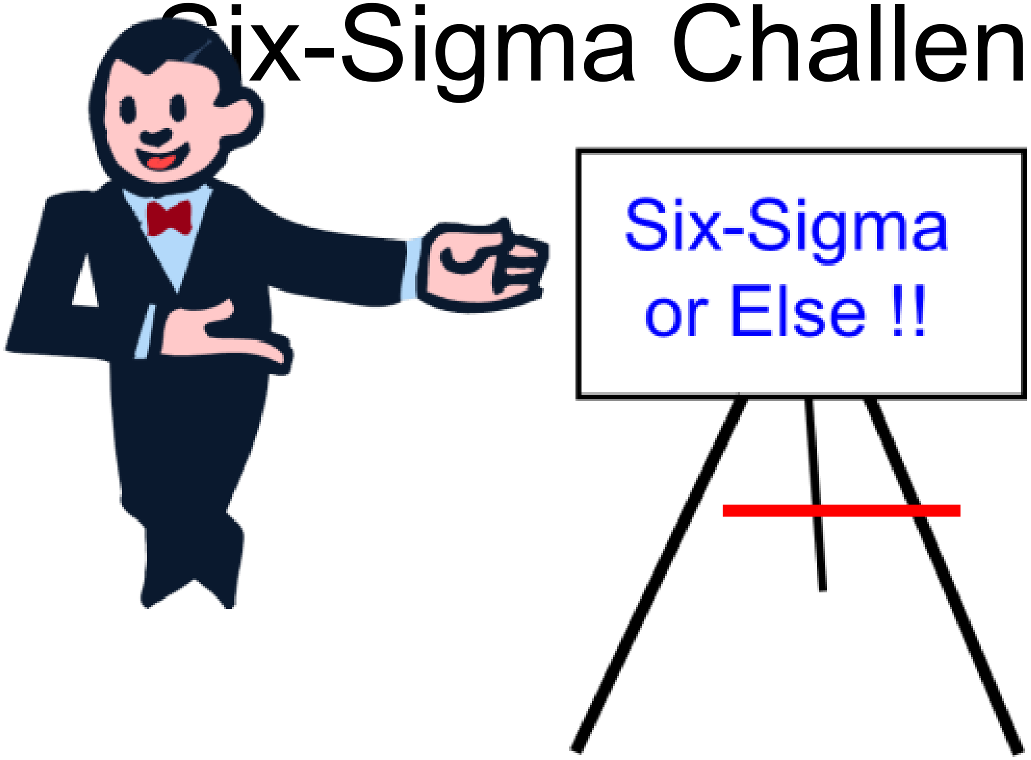
Many Companies Promote the Real Six-Sigma Challenge



From Wikipedia Sept 1 2021

Six Sigma (**6σ**) is a set of techniques and tools for process improvement. It was introduced by American engineer Bill Smith while working at Motorola in 1986.[1][2] A six sigma process is one in which 99.99966% of all opportunities to produce some feature of a part are statistically expected to be free of defects.

Many Companies Promote the Real Six-Sigma Challenge



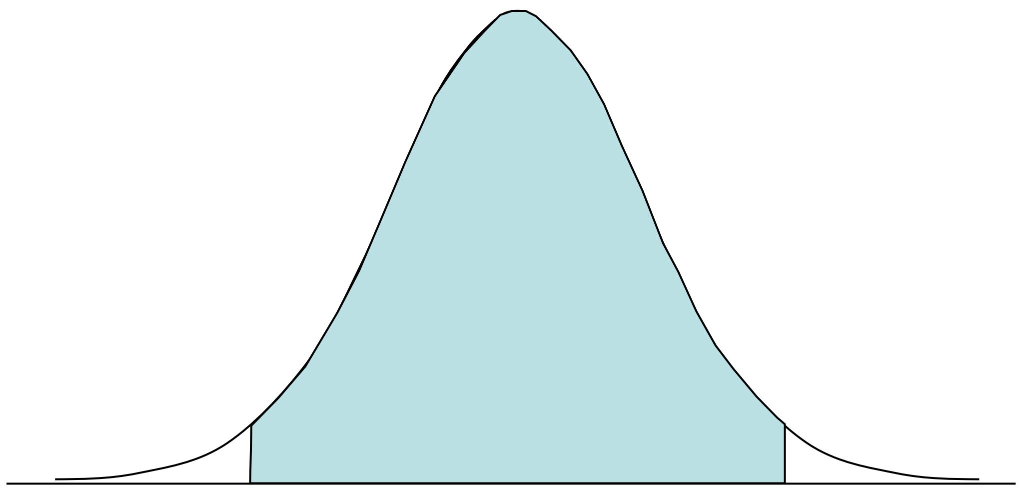
From Wikipedia Sept 1 2021

In 2005 Motorola attributed over $17 billion in savings to Six Sigma.[3]

By the late 1990s, about two-thirds of the Fortune 500 organizations had begun Six Sigma initiatives with the aim of reducing costs and improving quality.[6]

Yield at the Six-Sigma level

(Assume a Gaussian distribution)

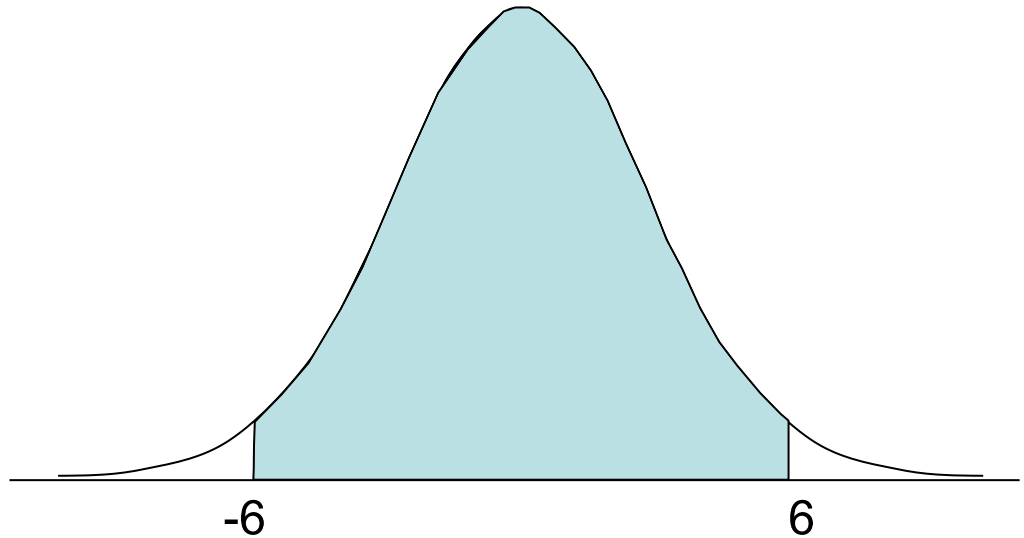


$$Y_{6sigma} = 2F_N(6) - 1$$

$$Y_{6sigma} = 0.9999999980$$

This is approximately 2 defects out of 1 billion parts

Yield at the Six-Sigma level



This is approximately 2 defects out of 1 billion parts

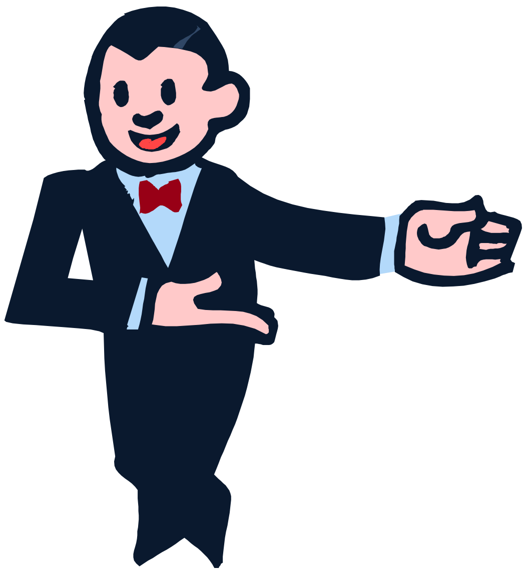
Would producing ICs with a yield at the six-sigma level be a good goal?

How about smart phones with defects at this level? (approx. 1.4B sold in 2020)

How about automobiles? (approx. 78 million produced in 2020)

Six-Sigma or Else !!

How serious is the “or Else” in the six-sigma programs?



Six-Sigma or Else !!

It is assumed that the performance or yield will drop, <u>for some reason</u>, by 1.5 sigma after a process has been established

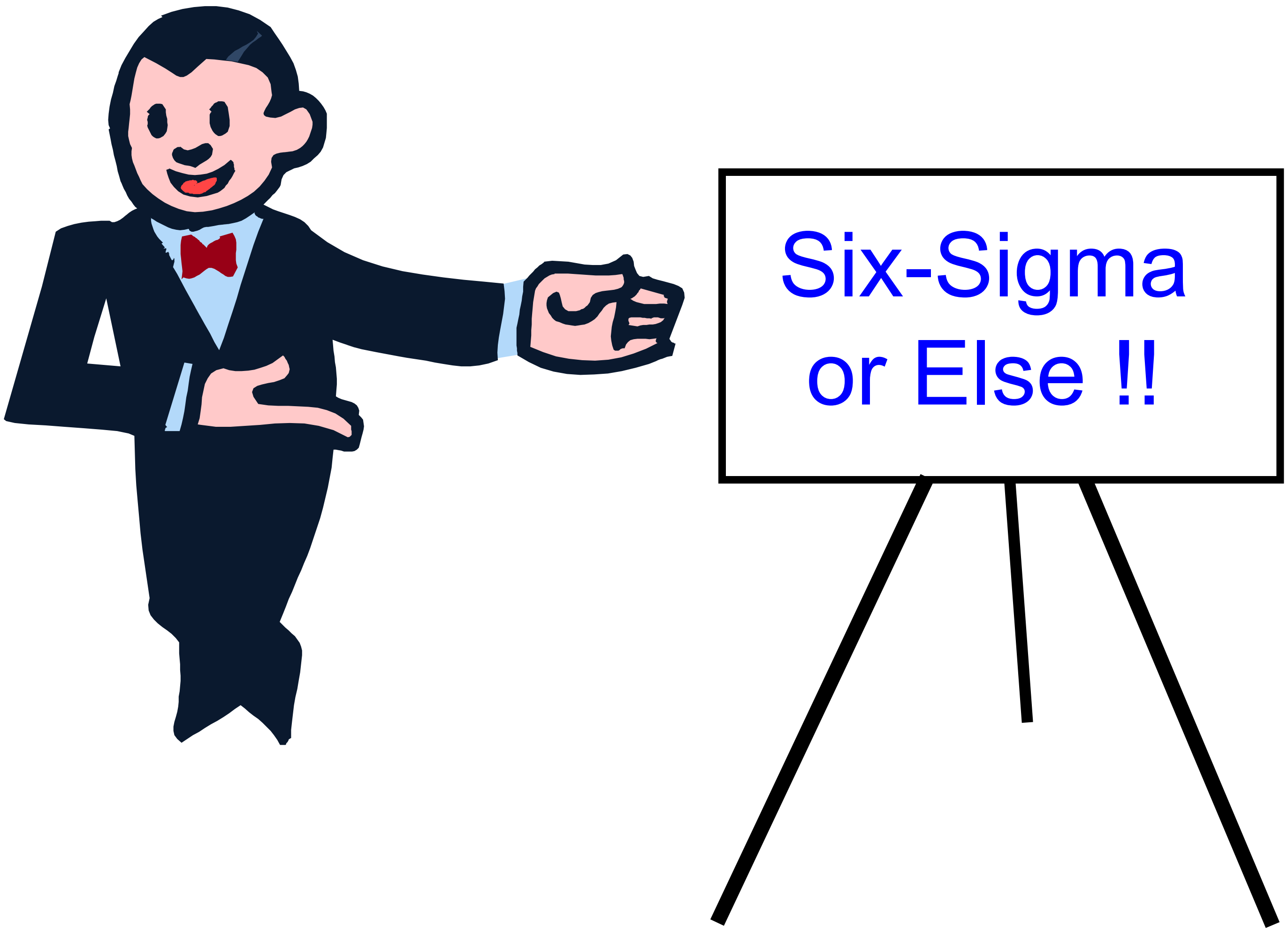
Initial "six-sigma" solutions really expect only 4.5 sigma performance in steady-state production

Assumption : Processes of interest are Gaussian (Normal)

4.5 sigma performance corresponds to 3.4 defects in a million

Observation: Any Normally distributed random variable can be mapped to a N(0,1) random variable by subtracting the mean and dividing by the variance

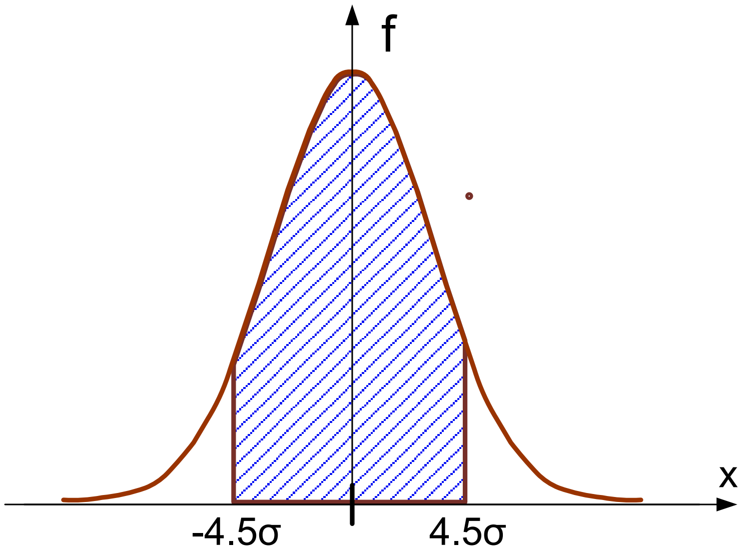
Meeting the Real Six-Sigma Challenge

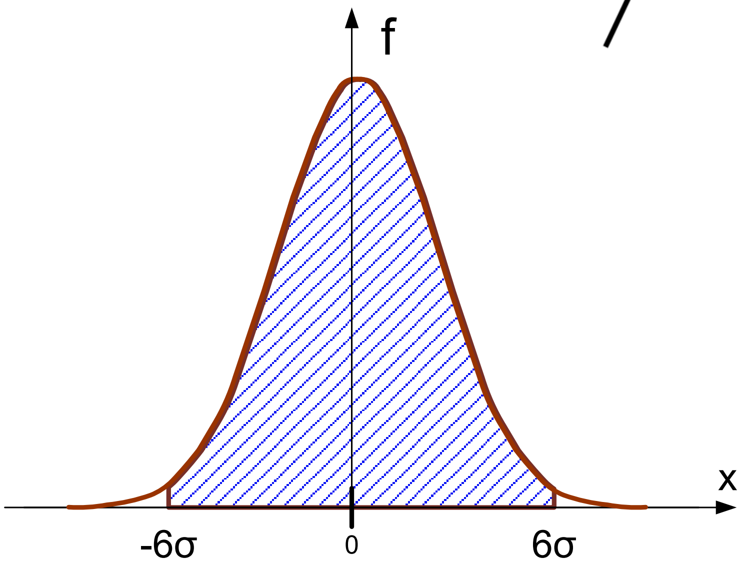


Highly Statistical Concept !

The Six-Sigma Challenge

Two-sided capability:





Long-term Capability

Tails are 6.8 parts in a million

Short-term Capability

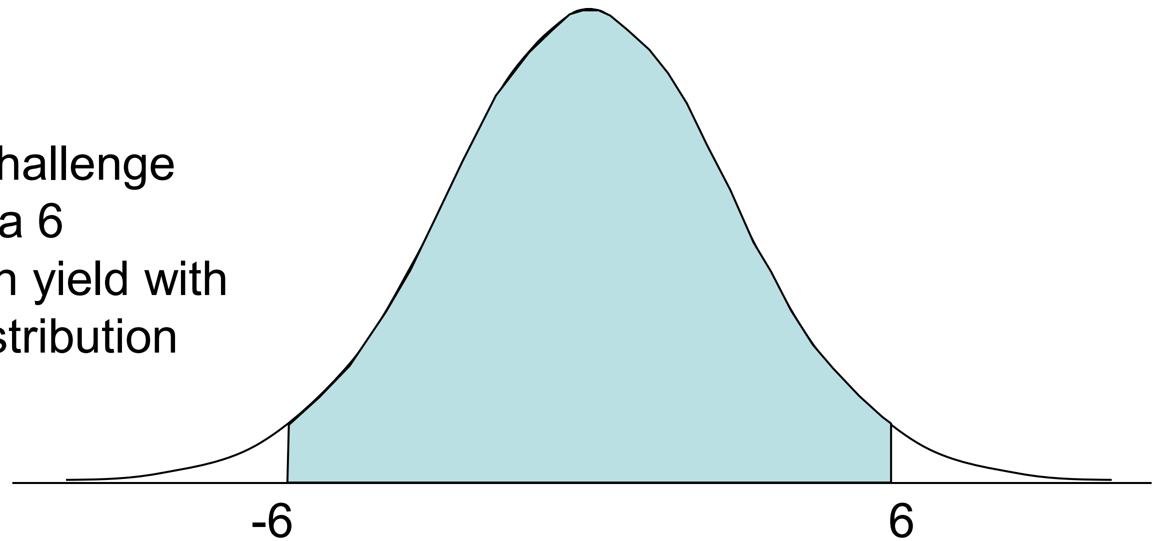
Tail is 2 parts in a billion

Six Sigma Performance is Very Good !!!

Example: Determine the maximum die area if the circuit yield is to initially meet the "six sigma" challenge for hard yield defects (Assume a defect density of $1cm^{-2}$ and only hard yield loss). Is it realistic to set six-sigma die yield expectations on the design and process engineers?

Solution:

The "six-sigma" challenge requires meeting a 6 standard deviation yield with a Normal (0,1) distribution



$$Y_{6sigma} = 2F_N(6) - 1$$

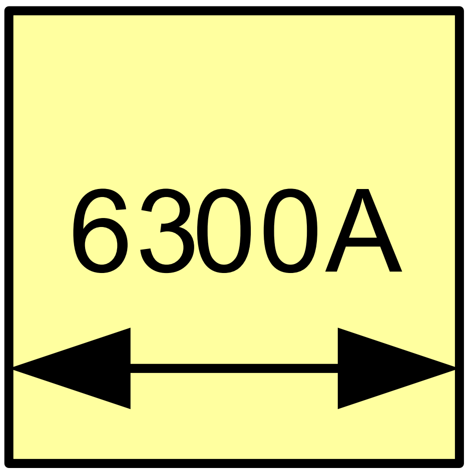
Recall: $F_N(6)=0.9999999980$

$$Y_{6sigma} = 0.999999996$$

Solution cont:

$$Y_H = e^{-Ad} \qquad A = \frac{-\ln(Y_H)}{d}$$

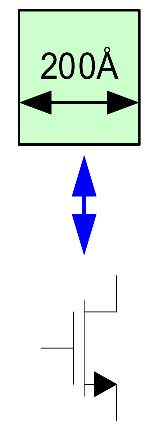
$$A = \frac{-\ln(.9999999980)}{1\text{cm}^{-2}} = 4.0E-9\text{cm}^2 = 40E6\,(\mathring{A})^2$$



Consider a 20nm process with
10x area overhead

$$A_{TRAN} = 10 * (200)^2\,(\mathring{A})^2 = 4E5\,(\mathring{A})^2$$

$$n = \frac{40E6(\mathring{A})^2}{4E5(\mathring{A})^2} = 100$$



This is comparable to the area required to fabricate about 100 minimum-sized transistors in a state of the art 20nm process

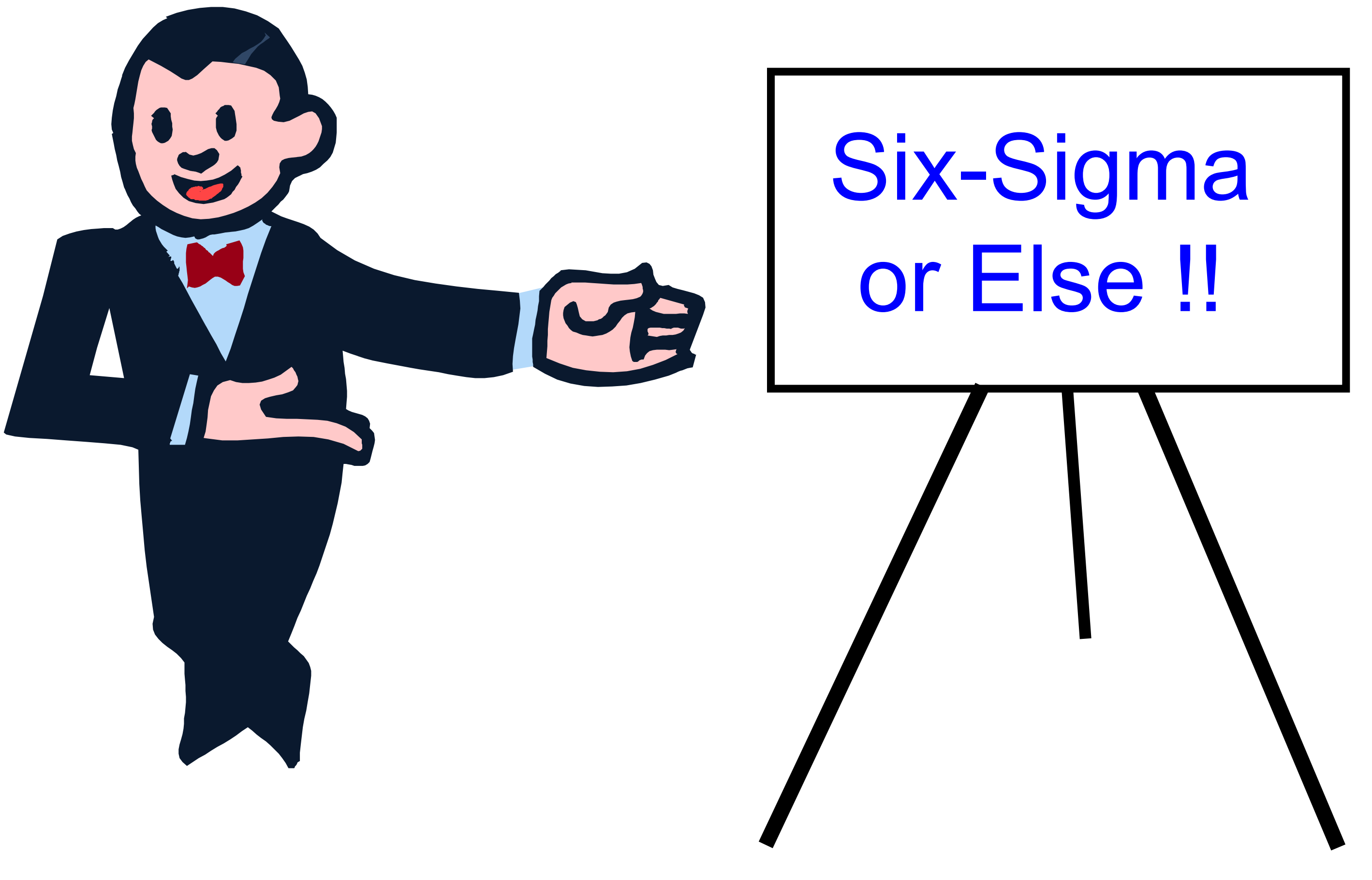
Solution cont:

Is it realistic to set six-sigma die hard yield expectations on the design and process engineers?

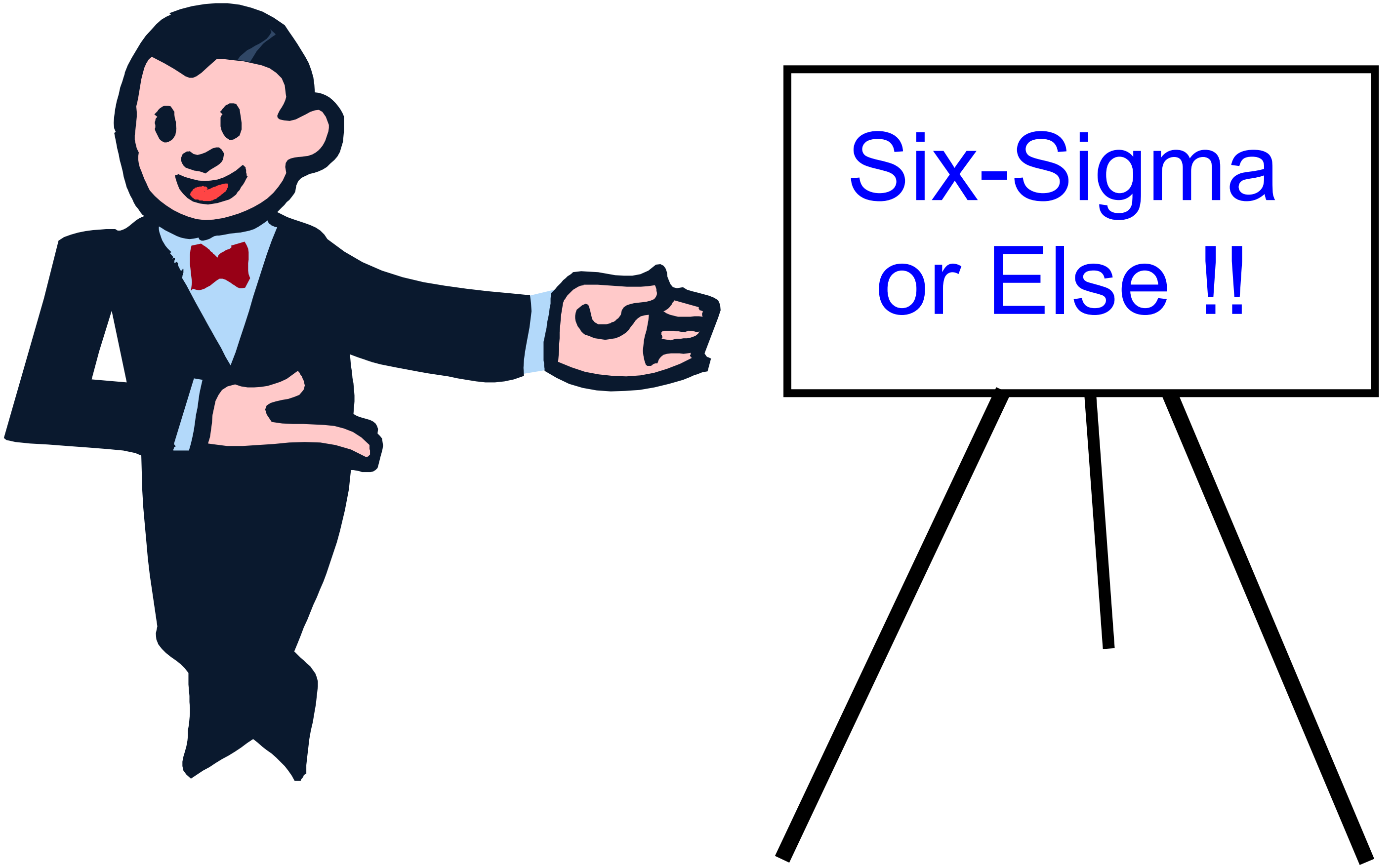
The best technologies in the world have orders of magnitude too many defects to build any useful integrated circuits with die yields that meet six-sigma performance requirements !!

Arbitrarily setting six-sigma design requirements will guarantee financial disaster !!

Meeting the Real Six-Sigma Challenge

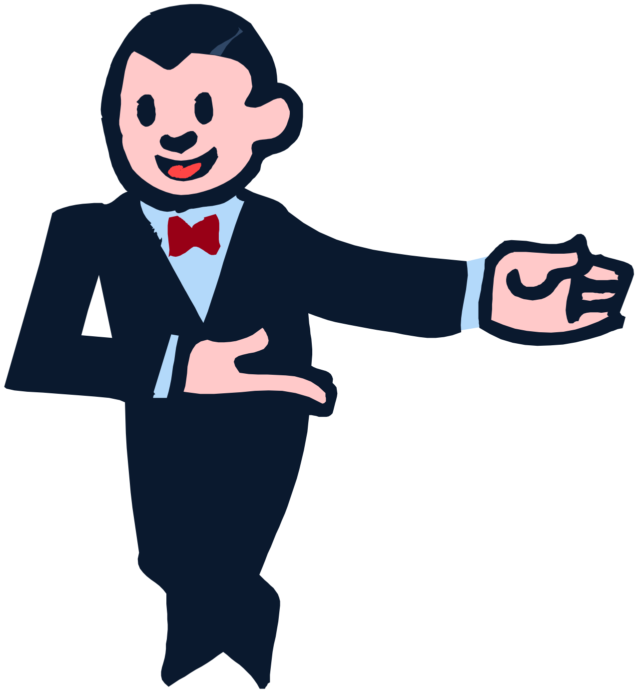


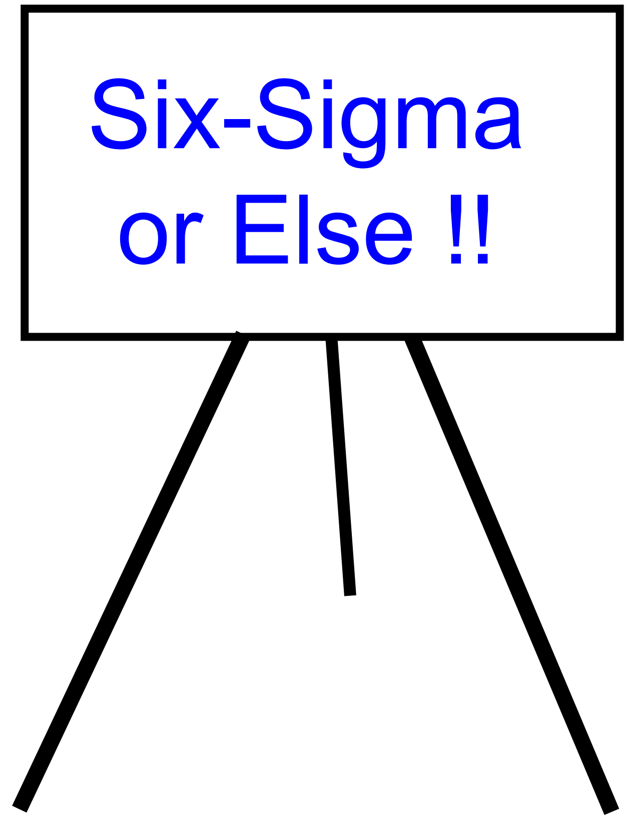
Meeting the Real Six-Sigma Challenge



Improving a yield by even one sigma often is VERY challenging !!

Meeting the Real Six-Sigma Challenge

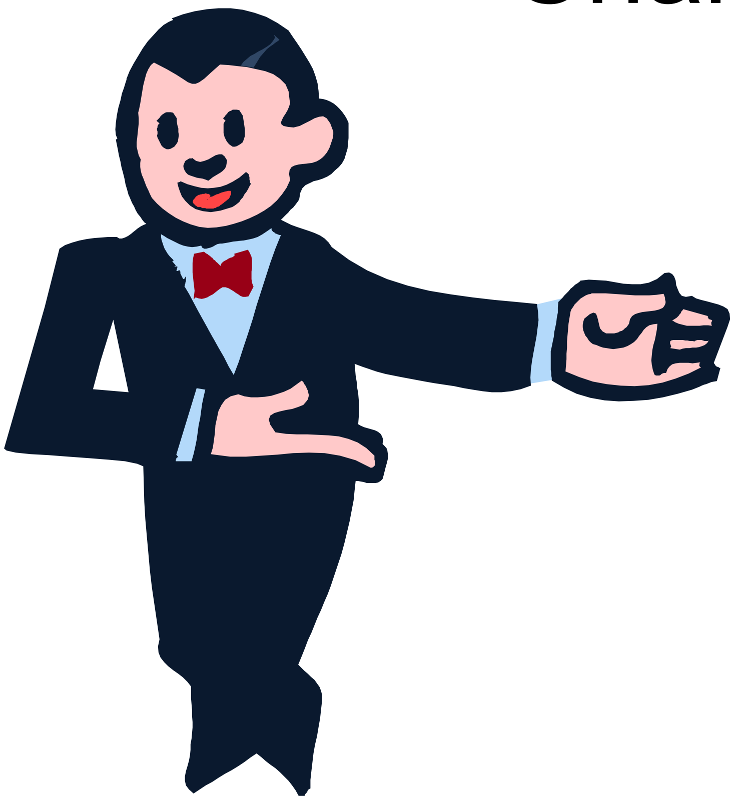


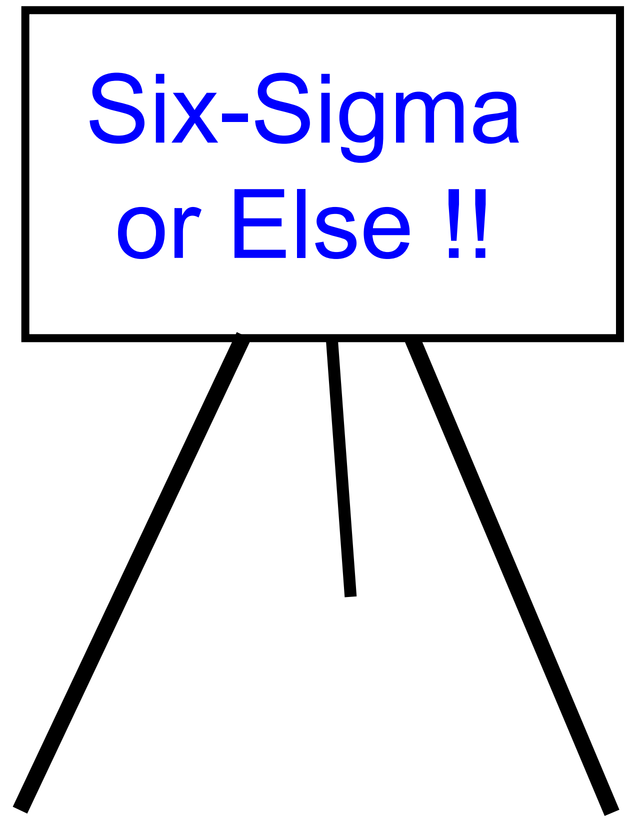


So, how has Motorola prospered with “meeting” the 6-sigma challenge?

MOTOROLA

Meeting the Real Six-Sigma Challenge

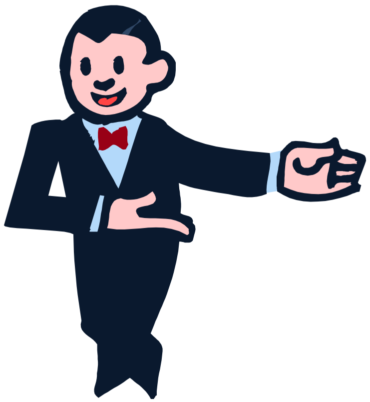




How has Motorola fared with the 6-sigma approach?

Motorola, Inc. (pronounced) was an American multinational[6] telecommunications company based in Schaumburg, Illinois, which was eventually divided into two independent public companies, Motorola Mobility and Motorola Solutions on January 4, 2011, after losing $4.3 billion from 2007 to 2009.[7]

Meeting the Real Six-Sigma Challenge



How has Motorola fared with the 6-sigma approach?

- Late 90's major competitor of Intel on microprocessors
- World leader in cell phones for a number of years
- Peaked at 150,000 employees
- ....

MOTOROLA

Former type	Public company
Industry	Telecommunications
Fate	Divided into Motorola Mobility and Motorola Solutions
Successor	Motorola Mobility Motorola Solutions
Founded	September 25, 1928
Defunct	January 4, 2011

- Sold military activities to General Dynamics 2000/2001
- Sold automotive products in 2006
- Spun off discrete components as ON semiconductor in 1999
- Spun off SPS as Freescale in 2003 (acquired by NXP in 2015)
- Sold Motorola Mobility to Google in 2011 (acquired by Lenovo in 2014)
- Motorola Solutions has 10,000 employees, down from over 150,000 at peak

Example: This was part of an article that appeared on Jan 26, 2022. The content of the article is not relevant but rather it serves as an example of use of statistics in our society

> The Washington Post in 2018 reported the statistical likelihood of a public school student in the US being killed by gunfire at school was roughly 1 in 614,000,000 since 1999.

Is this an abuse of statistics?

Statistics can be abused !

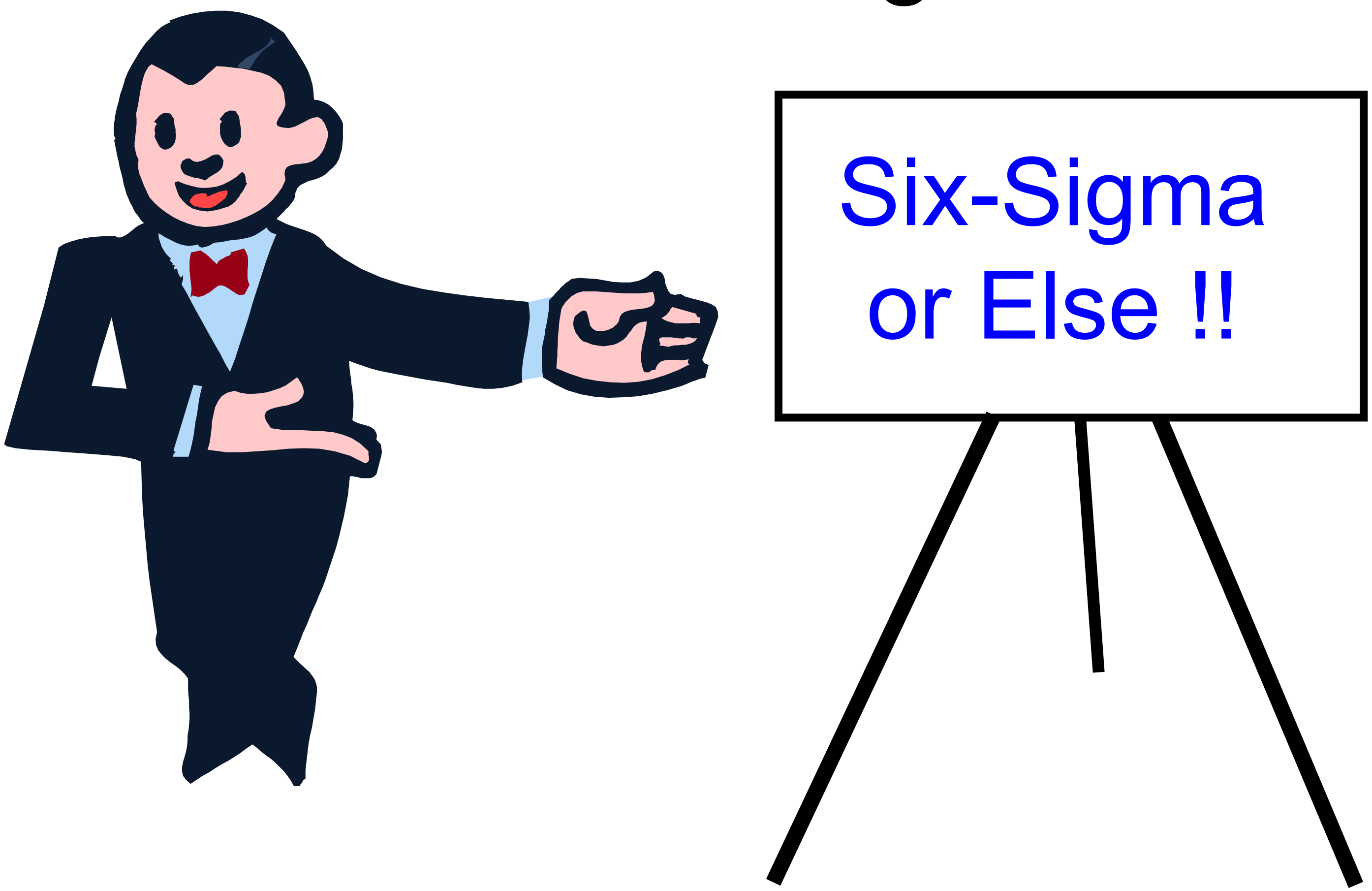
Many that are not knowledgeable incorrectly use statistics

Many use statistics to intentionally mislead the public

Some openly abuse statistics for financial gain or for manipulation purposes

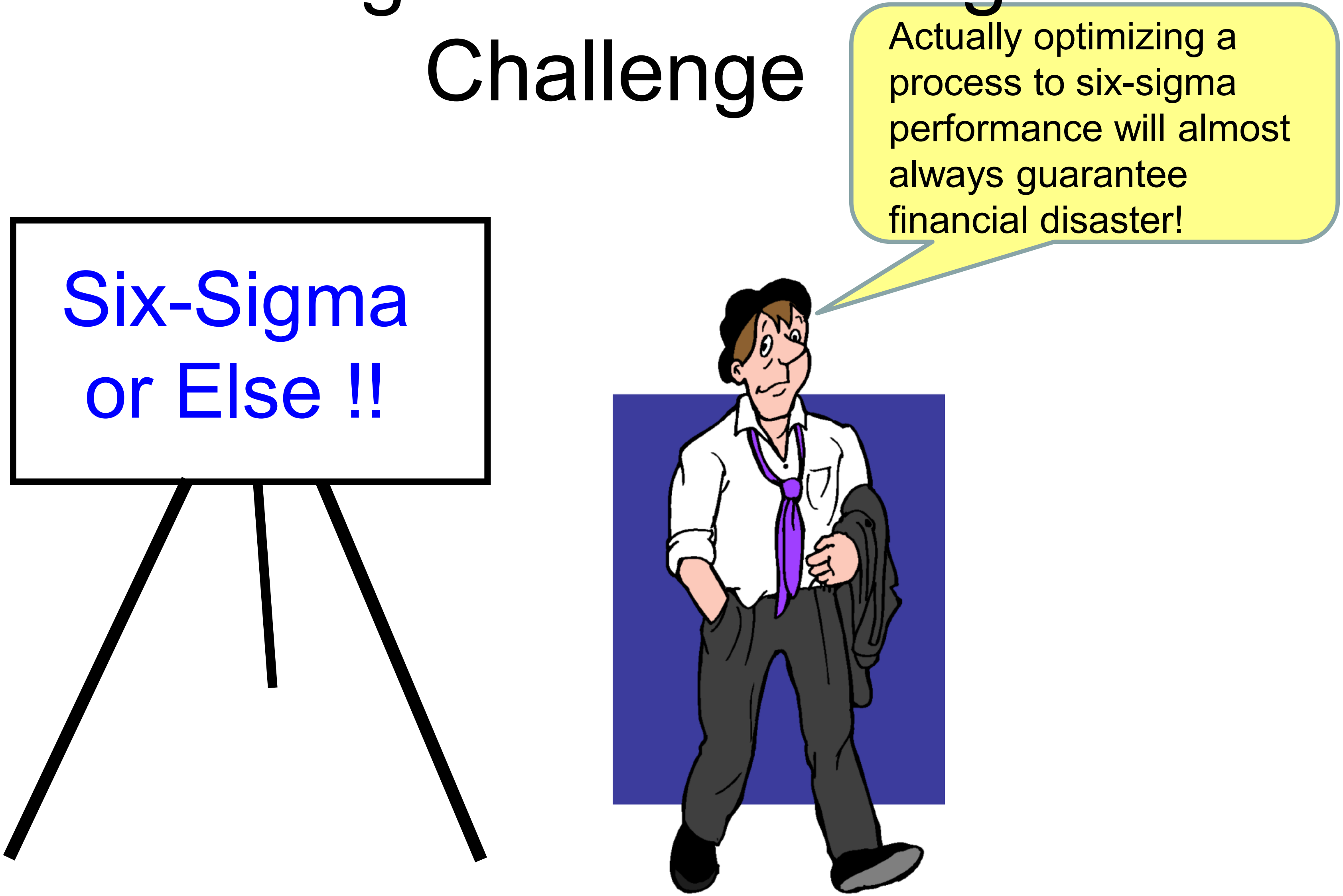
Keep an open mind to separate "good" statistics from "abused" statistics

Meeting the Real Six-Sigma Challenge

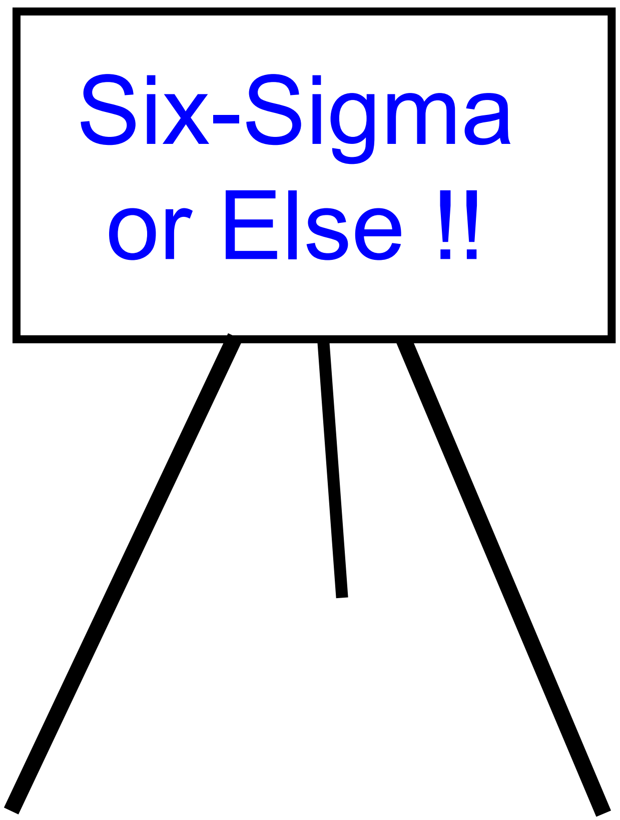


Six-sigma capability has almost nothing to do with optimizing profits and, if taken seriously, will likely **guarantee a financial fiasco** in most manufacturing processes

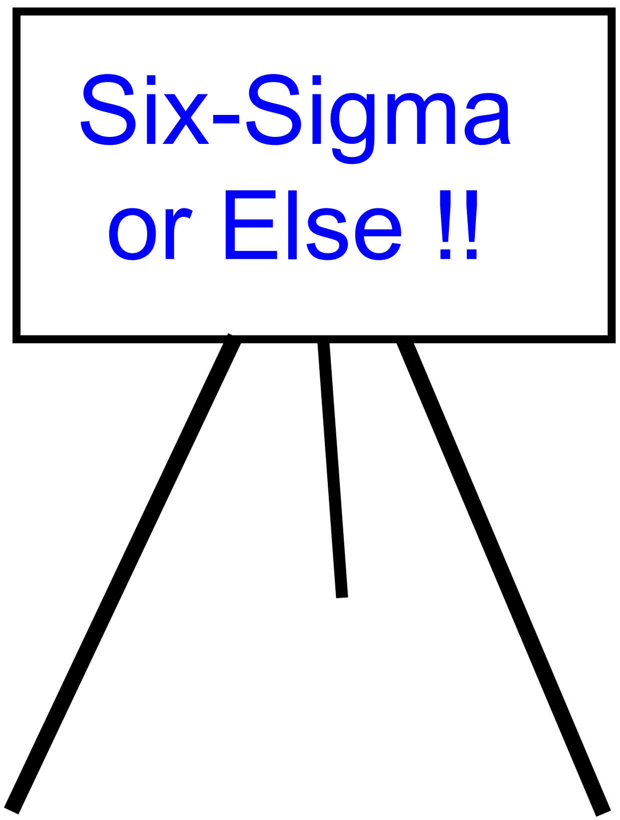
Meeting the real Six-Sigma Challenge



Meeting the real Six-Sigma Challenge

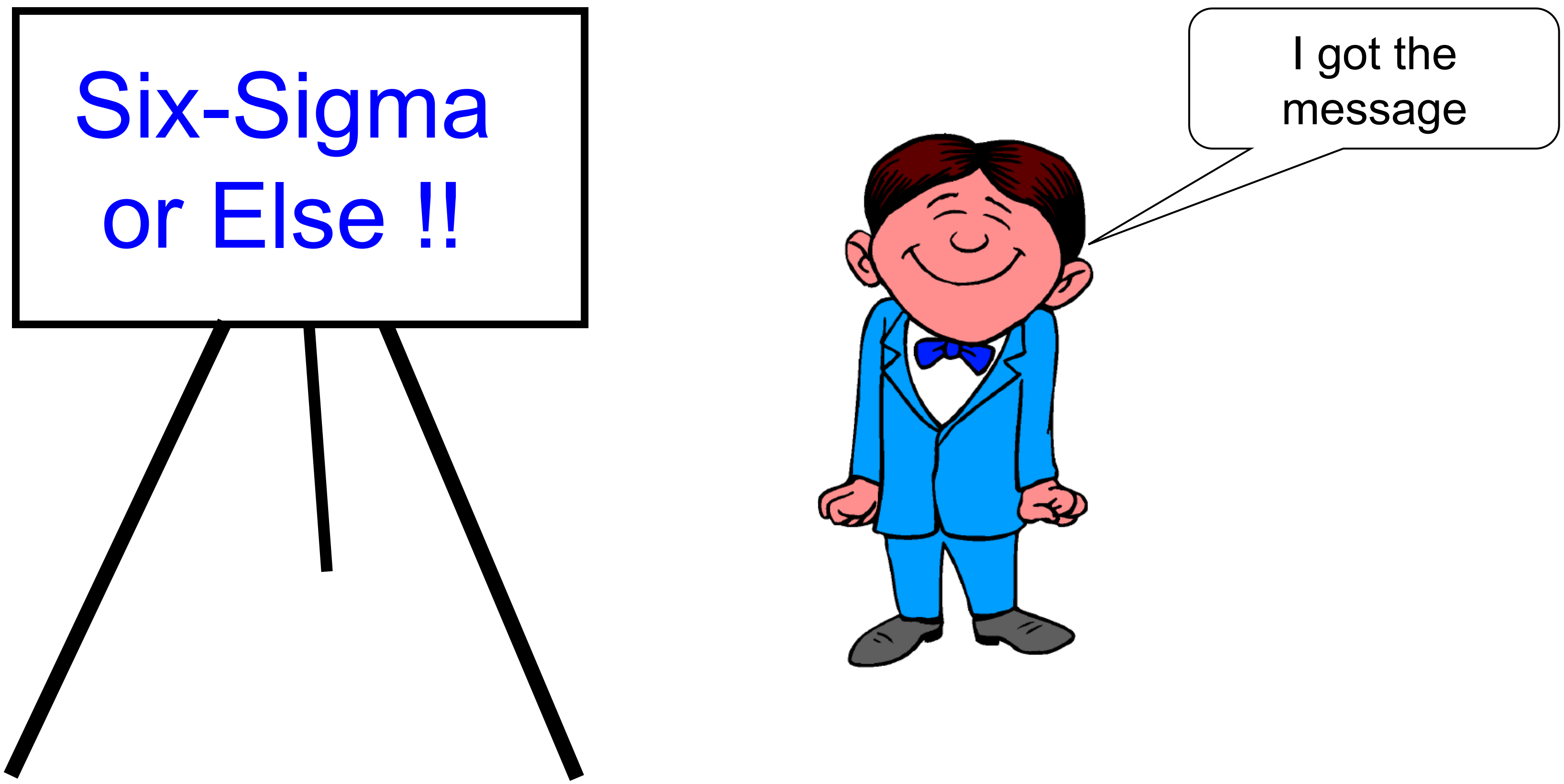


Meeting the real Six-Sigma Challenge

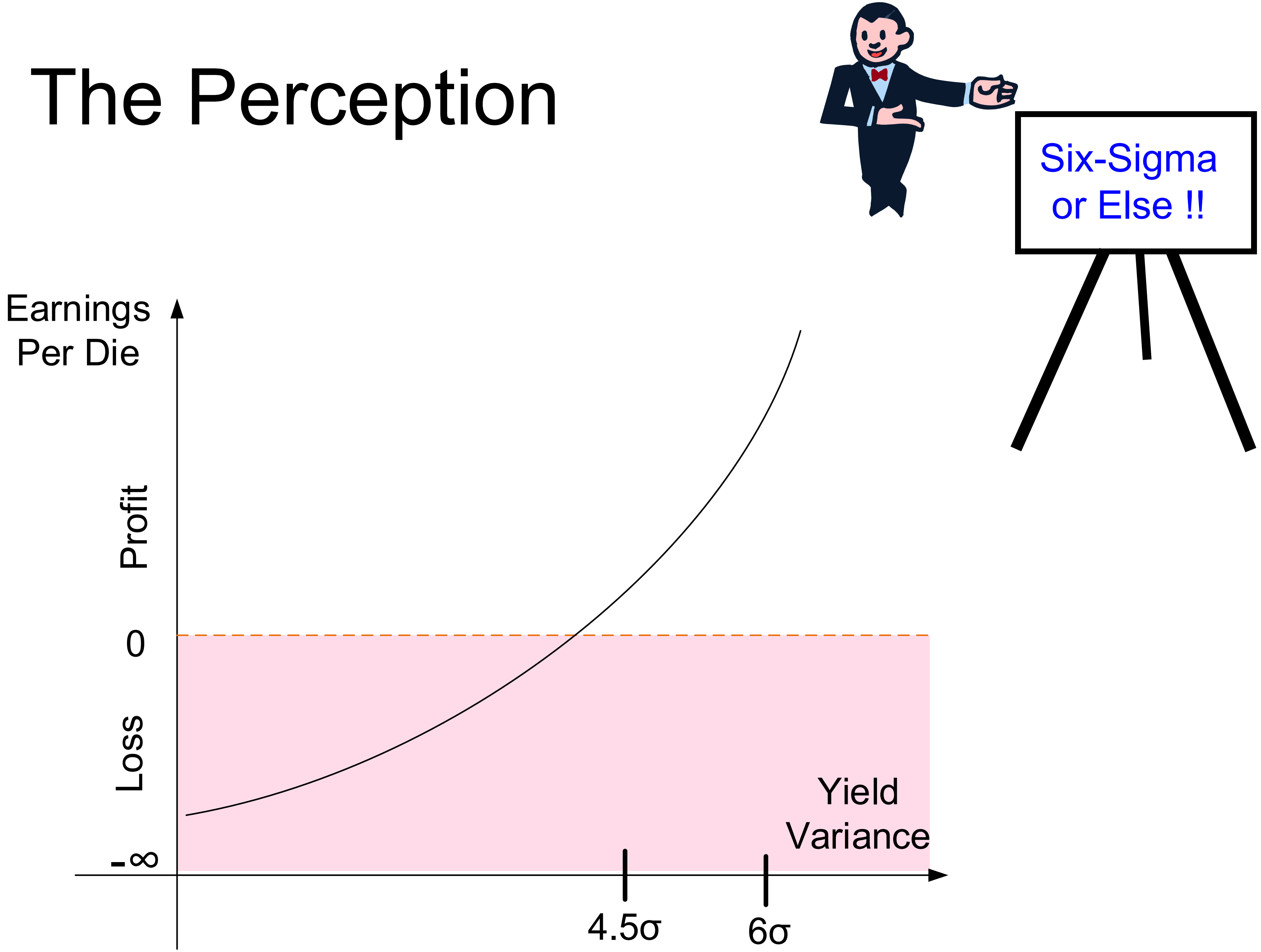


The concept of improving reliability (really profitability) is good – its just the statistics that are abused!

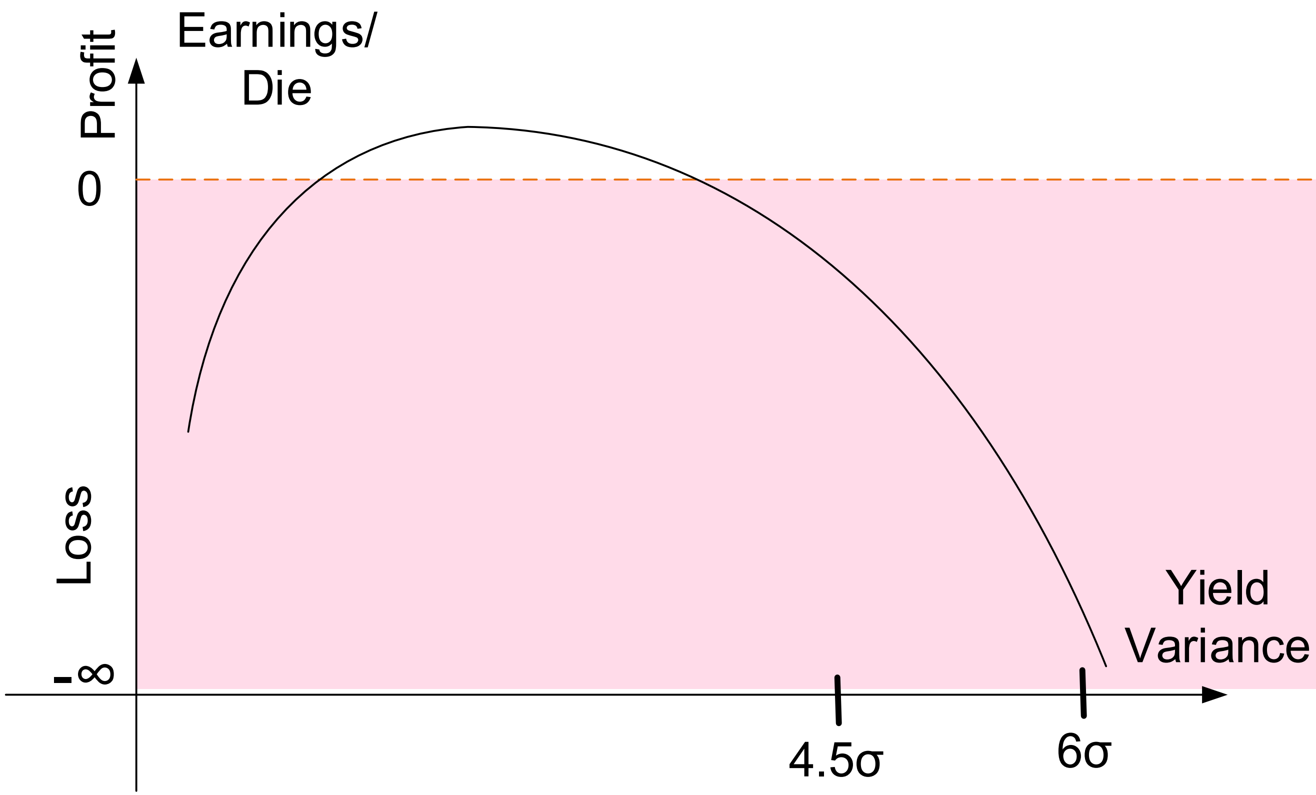
Meeting the real Six-Sigma Challenge



The Perception

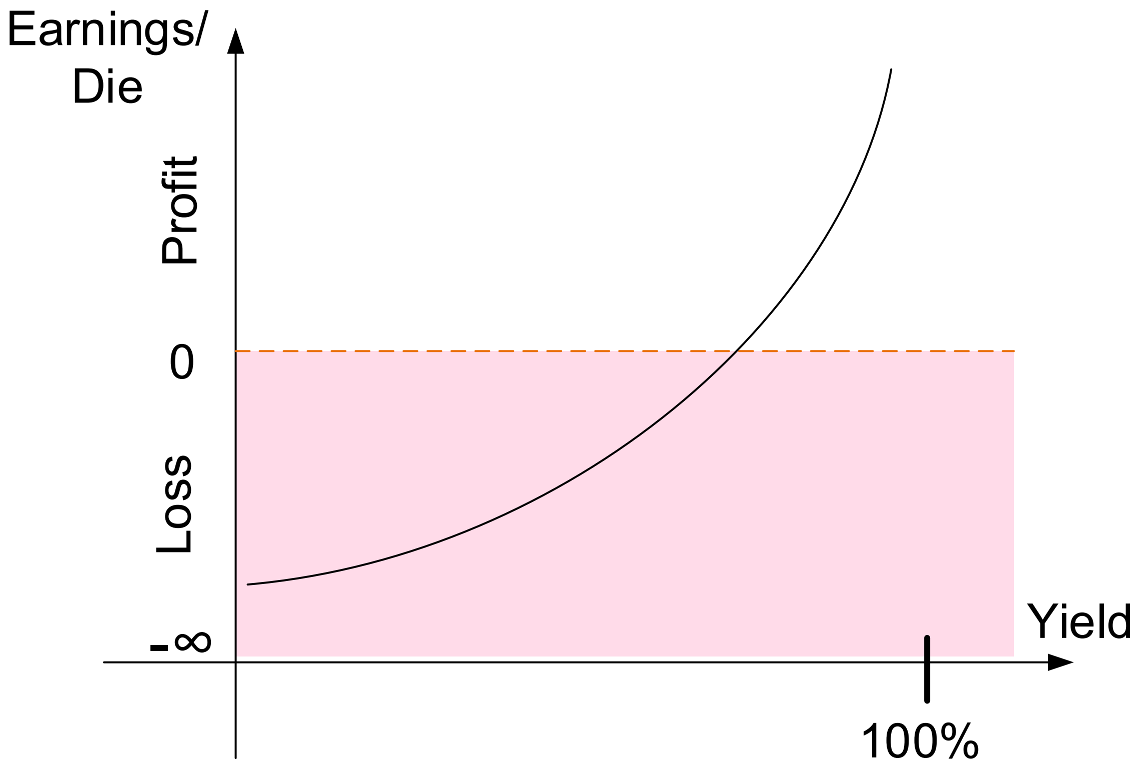


The Reality



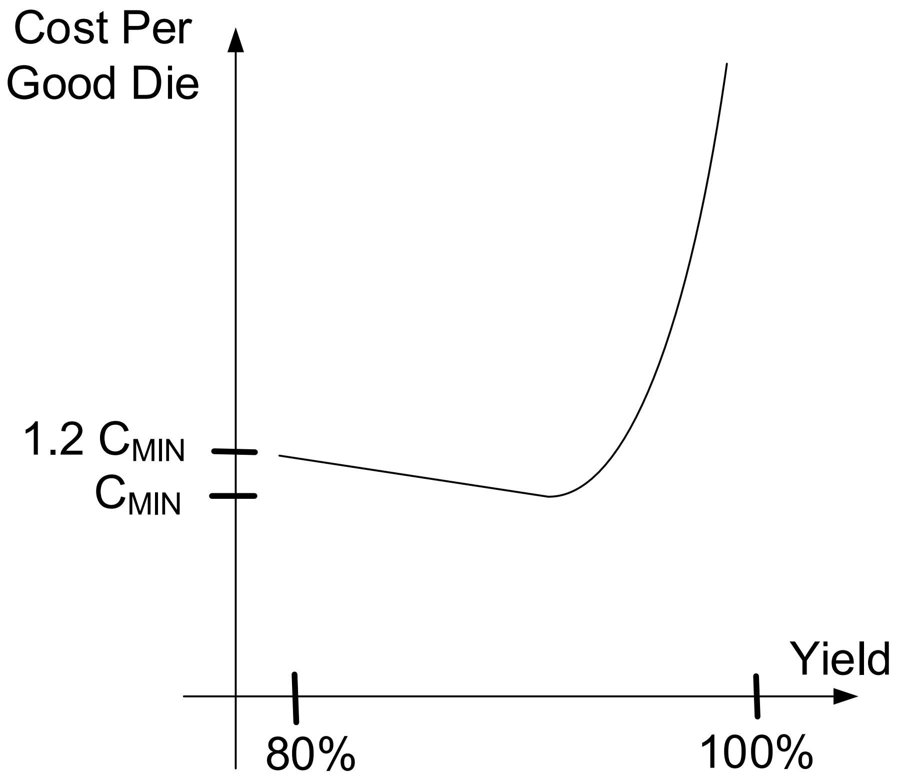
- Designing for 4.5σ or 6σ yield variance will almost always guarantee large losses
- Yield targets should be established to optimize earnings not yield variance

The Perception on Yield



Perception is often that goal should be to get yields as close to 100% as possible

The Reality about Yield



- Return on improving yield when yield is above 95% is small
- Inflection point could be at 99% or higher for some designs but below 50% for others
- Cost/good die will ultimately go to ∞ as yield approaches 100%

Designers goal should be to optimize profit, not an arbitrary yield target

Stay Safe and Stay Healthy !

End of Lecture 4